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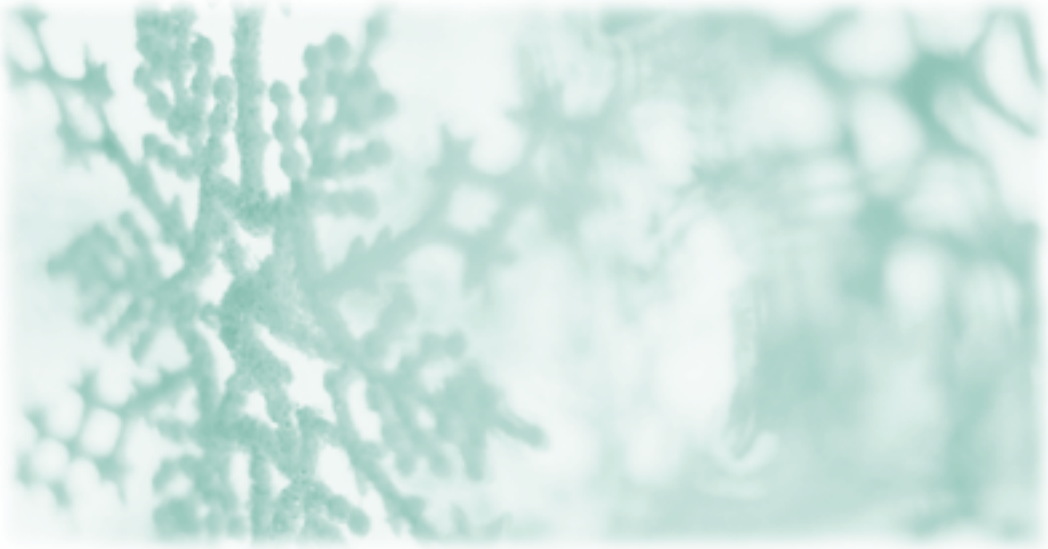
Profiles of Student Achievement in Mathematics at the TIMSS International Benchmarks:

U.S. Performance and Standards in an International Context



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Profiles of Student Achievement in Mathematics at the TIMSS International Benchmarks:
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The panel that developed the TIMSS benchmarks — Lillie Albert, John Dossey, William Hopkins, Chancey Jones, Mary Lindquist, Robert Garden, Barbara Japelj, David Robitaille, Graham Ruddock, and Hanako Senuma — played a critical role in the scale anchoring analysis. Without their expertise and the energy they devoted to reviewing the TIMSS items and drafting descriptions, this report would not have been possible. We also thank United States panel members, Lillie Albert, John Dossey, William Hopkins, Chancey Jones, and Mary Lindquist, for their work comparing the benchmarks with the NCTM Standards. Their expertise in mathematics education in the United States made the analysis possible. We also would like to thank all of the panel members, and in particular Mary Lindquist, for the time and energy they spent reviewing the results of the analyses.

Many people at the TIMSS International Study Center at Boston College played important roles in this project and were critical to the production of this report. Eugenio Gonzalez provided guidance in the analysis plans for the scale anchoring analysis and directed the programming for the scale anchoring analysis. Ce Shen constructed and managed the database and Isaac Li assisted with data entry and verification. Kathleen O'Connor worked with the mathematics panel in developing the benchmark descriptions and comparing the benchmarks with the NCTM Standards. Amanda Waterman prepared materials for the panel meetings. The analysis of the TIMSS science results was conducted in parallel with mathematics and thus the two companion reports benefited mutually from each other. We would like to specially acknowledge Teresa Smith, who coordinated the science report, for her contribution to this report. Thomas Hoffmann provided his creative expertise in the design and production of the cover and layout of the report.

CONTENTS

Introduction

The Third International Mathematics and Science Study (TIMSS) is the largest, most comprehensive, and most rigorous international study of schools and student achievement ever conducted. In 1995, more than 40 countries participated in an assessment of mathematics and science achievement at the fourth, eighth, and twelfth grades.[1] The results of the TIMSS 1995 assessment are available in a series of international reports published by the TIMSS International Study Center at Boston College.[2]

In 1998, the National Science Foundation awarded a grant for the TIMSS International Study Center to conduct an in-depth analysis of the TIMSS 1995 mathematics and science achievement results for fourth and eighth grades. The project had two components: (1) describe what students reaching the TIMSS international benchmarks of achievement know and can do in mathematics and science and begin to develop profiles of "world-class achievement," and (2) compare world-class mathematics and science achievement with U.S. national standards for these subjects.

1 In most countries, the grades tested for TIMSS were grades four, eight, and twelve.

2 Beaton, Martin, Mullis, Gonzalez, Smith & Kelly, 1996; Beaton, Mullis, Martin, Gonzalez, Kelly & Smith, 1996; Martin, Mullis, Beaton, Gonzalez, Smith & Kelly, 1997; Mullis, Martin, Beaton, Gonzalez, Kelly & Smith, 1997; and Mullis, Martin, Beaton, Gonzalez, Kelly, and Smith, 1998.

The purpose of the first part of the study was to interpret the TIMSS scale scores and analyze achievement at different points on the TIMSS scales. This was accomplished by conducting a scale anchoring analysis to describe achievement of students reaching four points on each of the TIMSS mathematics and science scales — the Top 10%, Upper Quarter, Median, and Lower Quarter international benchmarks (90th, 75th, 50th, and 25th international percentiles). Panels of mathematics and science educators examined the TIMSS items and identified what students reaching each benchmark know and can do.

For the second part of the study, the descriptions of performance at the benchmarks provided the basis for examining mathematics and science standards in the United States. The panels compared achievement at the benchmark levels with two prominent sets of standards for mathematics and science education — the National Council of Teachers of Mathematics' (NCTM) *Principles and Standards for School Mathematics* and the National Research Council's *National Science Education Standards* (NSES).[3]

This report presents detailed information about the mathematics achievement of fourth- and eighth-grade students in the countries that participated in TIMSS in 1995, and how that achievement aligns with the mathematics standards. A companion volume, *Profiles of Student Achievement in Science at the TIMSS International Benchmarks: U. S. Performance and Standards in an International Context*, presents the analysis of TIMSS science achievement in 1995 and compares it with the science standards.

3 National Council of Teachers of Mathematics (2000) and National Research Council (1995).

Overview

International Benchmarks of Mathematics Achievement

Overview — International Benchmarks

International
Benchmarks of Mathematics Achievement

As the United States continues to work to improve mathematics education, educators, curriculum developers, and policymakers need to know what students know and can do in mathematics, how this compares with students around the world, and what mathematics areas need more focus and effort. TIMSS provides detailed information about what students around the world know and can do in mathematics overall and in mathematics content areas. TIMSS also provides cross-national comparisons, enabling the United States to view its performance in an international context.

To provide benchmarks by which to compare countries' performance, achievement on TIMSS was reported at empirically derived benchmarks on the TIMSS scales — the Top 10% Benchmark, the Upper Quarter Benchmark, the Median Benchmark, and the Lower Quarter Benchmark. These mark the performance of the top 10%, top quarter, top half, and top three-quarters of students in the countries participating in TIMSS.[1]

1 The TIMSS international reports gave results for the 90th, 75th, and 50th international percentiles; these, as well as those for the 25th international percentiles, are included in this report.

To describe these benchmarks in terms of what students reaching them know and can do, the TIMSS International Study Center conducted an in-depth analysis to determine the mathematics content knowledge and understandings associated with each benchmark for the fourth and eighth grades. Together, the benchmark descriptions and the percentage of students in each country reaching each international benchmark show the strengths and weaknesses of fourth and eighth graders in the TIMSS countries. Moreover, by articulating performance at the TIMSS international benchmarks, "world class" achievement has been defined.

Developing Benchmarks of Mathematics Achievement

To develop descriptions of achievement at the TIMSS international benchmarks, TIMSS used the scale anchoring method.[2] Scale anchoring is a way of describing students' performance at different points on a scale in terms of what they know and can do. It involves a statistical component, in which items that discriminate between successive points on the scale are identified, and a judgmental component in which subject-matter experts examine the items and generalize to students' knowledge and understandings.

First, the TIMSS assessment results were used to identify the items that students reaching each international benchmark are likely to answer correctly and that students at the next lower benchmark are unlikely to answer correctly. Criteria were applied to group the items by benchmark level. For example, for the Top 10% benchmark, an item was included for the benchmark if at least 65 percent of students scoring at that scale point answered the item correctly and less than 50 percent of students scoring at the Upper Quarter benchmark answered the item correctly. Similarly, for the Upper Quarter benchmark, an item was included if at least 65 percent of students scoring at that point answered the item correctly and less than 50 percent of students at the Median benchmark answered the item correctly. The application of these criteria resulted in sets of items representing accomplishments of students reaching each benchmark.

Second, a panel of mathematics educators from the TIMSS countries examined the groups of items and summarized the content knowledge and conceptual understandings of the students

2 The analysis is fully documented in Kelly (1999).

reaching each level. The mathematics panel comprised ten individuals (see below) with extensive experience in mathematics education and a thorough knowledge of the TIMSS Curriculum Frameworks[3] and achievement tests.

The panelists' assignment consisted of three tasks: (1) work through the items one by one and describe what students answering each item correctly know and can do, or what they do to answer the item correctly; (2) based on the items for each benchmark, draft a detailed description of the knowledge, understandings, and skills demonstrated by students; and (3) select TIMSS example items to support and illustrate the benchmark descriptions.

Panel that Developed TIMSS International Benchmarks of Mathematics Achievement

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United States

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Mary Lindquist
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United States

Robert Garden
Consultant
New Zealand

Barbara Japelj
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Slovenia

David Robitaille
University of British Columbia
Canada

Graham Ruddock
National Foundation for Educational Research
England

Hanako Senuma
National Institute for Educational Research
Japan

* Now at the University of Texas at Austin.

3 | Robitaille, Schmidt, Raizen, McKnight, Britton, and Nicol (1993).

Fourth- and Eighth-Grade Performance at the TIMSS International Benchmarks of Mathematics Achievement

Three factors appear to distinguish performance at the four benchmarks: the mathematical operation required, the complexity of the numbers or number system, and the problem situation. At fourth grade, students scoring at the lower end of the scale demonstrate facility with whole numbers in simple problem situations, while their peers scoring at higher levels on the scale can use all four operations with whole numbers and can solve multi-step word problems. Students scoring at the Top 10% benchmark demonstrate an understanding of fractions and decimals and can perform simple division.

At eighth grade, students scoring at the lower end of the scale demonstrate an understanding of fractions and decimals and perform basic operations on decimal numbers, while students scoring at higher levels can perform basic operations on fractions, locate and use data in charts and graphs to solve problems, and have a grasp of beginning algebra. Students scoring at the Top 10% benchmark demonstrate that they can "bring things together." They organize information in problem-solving situations and apply relationships to solve problems.

Figures 1 and 2 show the summary descriptions of performance at the TIMSS international benchmarks of mathematics achievement for fourth and eighth grades. The figures also show the percentage of students in each country reaching each benchmark, with countries ranked by the percentage reaching the Top 10% benchmark. These descriptions of performance encapsulate the major accomplishments of students reaching each benchmark. The next two sections of this report provide the detailed descriptions of performance at each benchmark, with items illustrating what students reaching each benchmark know and can do.

Performance on the TIMSS scales is cumulative and the benchmark descriptions must be interpreted accordingly. That is, students reaching a particular benchmark demonstrate the knowledge and understandings characterizing that benchmark as well as the lower benchmarks. It is also important to recognize that some students scoring below a benchmark may indeed know or understand some of the concepts that characterize a higher level. For example, students scoring just below the scale score marking the Top 10% benchmark will have considerable success on the items for that benchmark. Similarly, students scoring above that scale score may not have success on all of the items for the Top 10% benchmark.

The performance of U.S. fourth graders was consistent with the international percentages, with 9 percent reaching the Top 10% benchmark, 26 percent reaching the Upper Quarter benchmark, 56 percent reaching the Median benchmark, and 83 percent reaching the Lower Quarter benchmark.

At eighth grade, U.S. students performed below the international percentages for the three highest benchmarks, with only 5 percent reaching the Top 10% benchmark, 18 percent reaching the Upper Quarter benchmark, 45 percent reaching the Median benchmark. Seventy-five percent of U.S. students reached the Lower Quarter benchmark, which matches the international percentage.

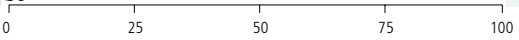
Interpreting Figures 1 and 2

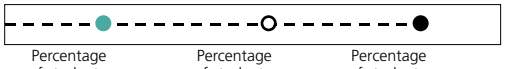
The percentages of students reaching the TIMSS benchmarks provide a way of interpreting differences in countries' performance on TIMSS. To illustrate, the data in Figure 1 show that 10 percent of all fourth graders in the countries participating in TIMSS achieved a score of 658 or better in mathematics. This score is thus the benchmark for the top 10 percent of fourth-grade students internationally. Similarly, 25 percent of all fourth graders achieved a score of 601 or better in mathematics, and this is the Upper Quarter benchmark, and so on. If all countries had the same performance, then in each country 10 percent of the students would be at or above the Top 10% benchmark, 25 percent would be at or above the Upper Quarter benchmark, half would be at or above the Median benchmark, and 75 percent would be at or above the Lower Quarter benchmark. While some countries achieved nearly this pattern, there was wide variation in the percentages of students reaching the benchmarks. For example, in mathematics at the fourth grade (Figure 1), in top-performing Singapore, 39 percent of fourth-grade students reached the Top 10% international benchmark. In other words, nearly half of the students in Singapore performed as well as the top 10 percent of students across all TIMSS countries. In contrast, 0 percent of students in Kuwait and Iran reached this benchmark.

FIGURE 1

Percentages of Students Reaching TIMSS International Benchmarks of Mathematics Achievement — Fourth Grade*

	Percentages of Students Reaching International Benchmarks	Top 10%	Upper Quarter	Median	Lower Quarter
Singapore		39 (2.3)	62 (2.0)	82 (1.2)	92 (0.7)
Korea		26 (1.2)	56 (1.3)	85 (0.8)	97 (0.3)
Japan		23 (0.9)	48 (1.2)	79 (0.8)	94 (0.4)
Hong Kong		18 (1.5)	44 (2.5)	76 (1.8)	93 (0.8)
Czech Republic		15 (1.3)	34 (1.6)	64 (1.4)	88 (0.7)
Netherlands		13 (1.1)	36 (1.9)	72 (1.8)	95 (0.7)
Australia		12 (0.7)	27 (1.2)	55 (1.4)	82 (1.2)
Hungary		11 (1.1)	27 (1.7)	56 (1.8)	83 (1.1)
Austria		11 (1.1)	31 (1.5)	63 (1.6)	88 (1.0)
Slovenia		11 (0.9)	28 (1.4)	58 (1.7)	86 (1.2)
Ireland		10 (0.7)	28 (1.6)	59 (1.8)	85 (1.2)
United States		9 (0.8)	26 (1.3)	56 (1.6)	83 (0.9)
Canada		7 (0.8)	21 (1.4)	49 (1.5)	80 (1.3)
England		7 (0.7)	17 (1.2)	39 (1.5)	71 (1.3)
Scotland		6 (0.8)	18 (1.4)	43 (1.8)	74 (1.5)
Israel		6 (0.7)	21 (1.5)	49 (1.9)	80 (1.2)
Latvia (LSS)		6 (1.3)	19 (2.1)	44 (2.3)	75 (1.7)
Cyprus		4 (0.5)	12 (0.9)	36 (1.5)	67 (1.4)
New Zealand		3 (0.7)	12 (1.1)	36 (2.0)	66 (1.9)
Greece		3 (0.5)	11 (0.9)	32 (1.8)	63 (1.9)
Norway		2 (0.3)	8 (0.8)	33 (1.6)	70 (1.6)
Iceland		1 (0.3)	5 (0.7)	20 (1.3)	55 (1.7)
Portugal		1 (0.2)	6 (0.6)	24 (1.3)	56 (1.7)
Thailand		1 (0.2)	5 (1.0)	27 (2.4)	65 (2.6)
Iran, Islamic Rep.		0 (0.1)	1 (0.4)	7 (1.3)	29 (2.3)
Kuwait		0 (0.1)	0 (0.1)	3 (0.4)	17 (1.0)





Percentage of students at or above **Top 10%** Benchmark

Percentage of students at or above **Upper Quarter** Benchmark

Percentage of students at or above **Median** Benchmark

Top 10% Benchmark (90th Percentile) = 658

Upper Quarter Benchmark (75th Percentile) = 601

Median Benchmark (50th Percentile) = 535

Lower Quarter Benchmark (25th Percentile) = 464

The international benchmarks correspond to the percentiles computed from the combined data from all of the countries participating in 1995.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1994-1995.

* Fourth grade in most countries.

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Summary of Performance at TIMSS International Benchmarks
Mathematics — Fourth Grade

Top 10% International Benchmark

Add/subtract decimal numbers in word problems; convert fractions to decimal numbers; understand verbal and pictorial representations of decimals; understand place value to first decimal place; solve two-step problems involving multiplication and division (one-digit); use simple proportional reasoning to solve problems; relate units of measurement; determine the relationship, involving division, between pairs of whole numbers.

Upper Quarter International Benchmark

Solve multi-step word problems involving addition, subtraction, and multiplication of whole numbers; solve simple rate and ratio problems; use understanding of place value to solve problems; use information in tables and graphs to solve problems; locate a point on a grid; identify numerical expressions and rules involving multiplication.

Median International Benchmark

Add, subtract, and multiply whole numbers with regrouping; recognize familiar fractions; add/subtract decimals; demonstrate familiarity with units of mass; read, locate, and compare data in tables and graphs; recognize and use lines of symmetry; recognize and complete number sentences with addition, subtraction, multiplication.

Lower Quarter International Benchmark

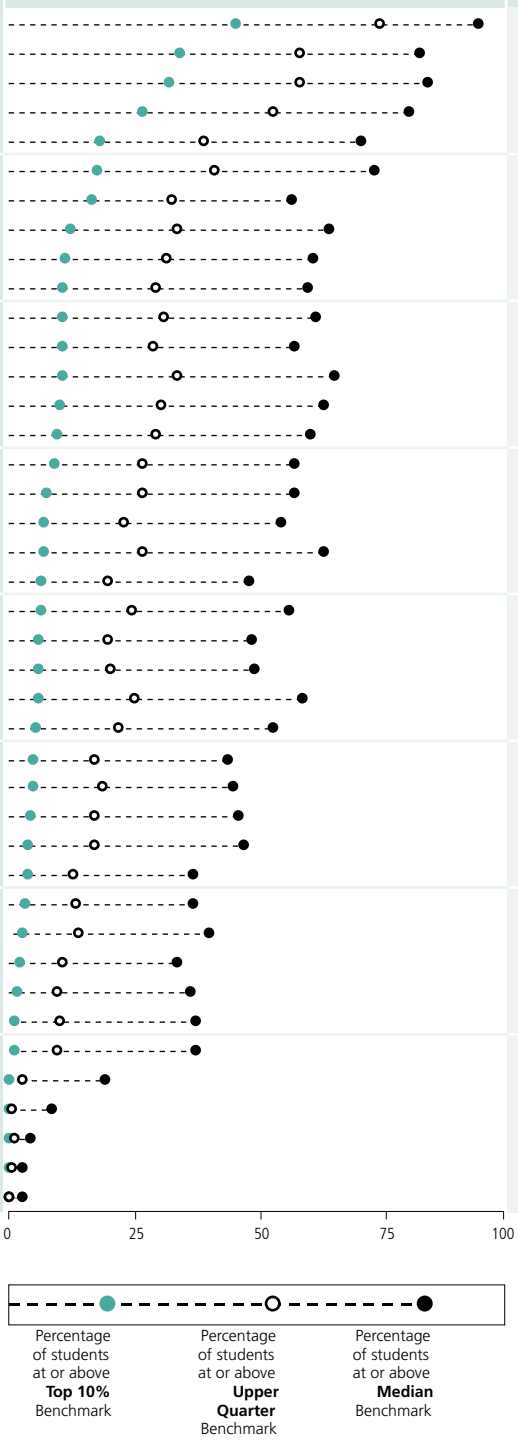
Name, order, and use four-digit numbers in a variety of representations; add whole numbers; multiply small whole numbers without regrouping; read and locate information in simple graphs; recognize representations of one-half, triangles, and simple patterns.

FIGURE 1

FIGURE 2

Percentages of Students Reaching TIMSS International Benchmarks of Mathematics Achievement — Eighth Grade*

	Top 10%	Upper Quarter	Median	Lower Quarter
Singapore	45 (2.5)	74 (2.1)	94 (0.8)	99 (0.2)
Korea	34 (1.1)	58 (1.0)	82 (0.8)	93 (0.5)
Japan	32 (0.8)	58 (0.9)	83 (0.6)	95 (0.3)
Hong Kong	27 (2.1)	53 (2.6)	80 (2.4)	92 (1.3)
Czech Republic	18 (1.9)	39 (2.3)	70 (1.9)	92 (0.6)
Belgium (Fl)	17 (1.2)	41 (2.3)	73 (2.9)	92 (1.5)
Bulgaria	16 (1.9)	33 (2.7)	57 (2.7)	83 (1.6)
Slovak Republic	12 (1.0)	33 (1.5)	64 (1.6)	88 (0.8)
Austria	11 (0.7)	31 (1.3)	61 (1.4)	86 (1.1)
Hungary	11 (0.8)	29 (1.3)	60 (1.6)	86 (1.1)
Slovenia	11 (0.7)	31 (1.4)	61 (1.5)	88 (0.8)
Australia	11 (0.9)	29 (1.5)	57 (1.7)	83 (1.1)
Switzerland	11 (0.7)	33 (1.2)	65 (1.4)	89 (1.0)
Netherlands	10 (1.6)	30 (2.7)	63 (3.2)	87 (2.3)
Russian Federation	10 (0.7)	29 (2.4)	60 (2.6)	85 (1.4)
Ireland	9 (1.0)	27 (1.9)	57 (2.4)	83 (1.7)
Canada	7 (0.7)	25 (1.1)	58 (1.2)	85 (0.9)
Thailand	7 (1.2)	23 (2.6)	54 (2.7)	85 (1.4)
France	7 (0.8)	26 (1.5)	63 (1.5)	92 (0.8)
England	7 (0.6)	20 (1.1)	48 (1.4)	77 (1.2)
Israel	6 (0.9)	24 (2.5)	56 (2.6)	83 (1.7)
New Zealand	6 (0.8)	20 (1.6)	48 (2.2)	78 (1.5)
Germany	6 (0.7)	24 (1.7)	49 (2.3)	79 (1.5)
Belgium (Fr)	6 (0.6)	25 (1.5)	58 (1.7)	85 (1.4)
Sweden	5 (0.5)	22 (1.2)	53 (1.5)	84 (1.1)
Scotland	5 (0.9)	17 (2.1)	44 (2.7)	75 (1.9)
United States	5 (0.6)	18 (1.5)	45 (2.3)	75 (1.6)
Norway	4 (0.4)	17 (0.9)	46 (1.2)	79 (0.9)
Denmark	4 (0.5)	17 (1.0)	47 (1.6)	78 (1.3)
Greece	3 (0.4)	13 (0.8)	37 (1.5)	68 (1.2)
Romania	3 (0.4)	13 (1.1)	36 (2.0)	67 (1.8)
Latvia (LSS)	3 (0.5)	14 (1.2)	40 (1.5)	75 (1.4)
Cyprus	2 (0.3)	11 (0.6)	34 (1.1)	64 (1.1)
Spain	2 (0.2)	10 (0.7)	36 (1.2)	75 (1.0)
Iceland	1 (0.3)	10 (1.3)	37 (2.9)	75 (2.2)
Lithuania	1 (0.3)	10 (1.0)	34 (1.8)	69 (1.8)
Portugal	0 (0.1)	2 (0.4)	19 (1.3)	58 (1.5)
Iran, Islamic Rep.	0 (0.0)	0 (0.2)	9 (0.8)	43 (1.8)
Colombia	0 (0.0)	1 (0.3)	4 (0.8)	20 (1.7)
Kuwait	0 (0.0)	0 (0.1)	3 (0.5)	21 (1.6)
South Africa	0 (0.0)	0 (0.0)	3 (0.9)	10 (2.3)



Top 10% Benchmark (90th Percentile) = 656

Upper Quarter Benchmark (75th Percentile) = 587

Median Benchmark (50th Percentile) = 509

Lower Quarter Benchmark (25th Percentile) = 425

The international benchmarks correspond to the percentiles computed from the combined data from all of the countries participating in 1995.

*Eighth grade in most countries.

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1994-1995.

FIGURE 2

Summary of Performance at TIMSS International Benchmarks
Mathematics — Eighth Grade

Top 10% International Benchmark

Organize information in problem-solving situations; solve time-distance-rate problems involving conversion of measures within a system; apply relationships – fractions and decimals, ratios, properties of geometric figures, and algebraic rules – to solve problems; solve word problems involving the percentage of increase.

Upper Quarter International Benchmark

Order, relate, multiply, and divide fractions and decimals; relate area and perimeter; understand simple probability; use knowledge of geometric properties to solve problems; identify algebraic expressions and solve equations with two variables.

Median International Benchmark

Use understanding of rounding in problem situations; perform basic operations with familiar fractions; understand place value of decimal numbers; understand measurement in several settings; locate data in charts and graphs to solve word problems; know and use simple properties of geometric figures to solve problems; identify algebraic expressions and solve equations with one variable.

Lower Quarter International Benchmark

Understand different representations of fractions – verbal and decimal; add and subtract decimals with the same number of decimal places; read, locate, and compare data in charts and graphs; calculate average of whole numbers.

FIGURE 2

Comparing TIMSS Benchmarks with NCTM Standards

In the last decade, a great deal of work has been done in the United States at the national, state, and local levels to reform mathematics education and establish clear and high standards for performance. Since 1989, when the National Council of Teachers of Mathematics (NCTM) published *Curriculum and Evaluation Standards for School Mathematics* (hereafter referred to as the *Standards*), the mathematics education community has had the benefit of a unified set of goals for mathematics teaching and learning. The standards adopted by the NCTM have served as curriculum standards in their own right and have been a springboard for state and local efforts to focus and improve mathematics teaching. Many states and districts have enhanced their mathematics programs or developed their own curricula to address the NCTM *Standards*.

During the last five years, the NCTM has updated the 1989 *Standards* to reflect the experience of practitioners who used them during the past decade, changes in technology available for teaching mathematics, and the latest research on mathematics teaching and learning. The NCTM published *Principles and Standards for School Mathematics* (hereafter referred to as *Principles and Standards*) in April 2000.

Principles and Standards contains a set of principles for school mathematics and "Content Standards" and "Process Standards" for pre-kindergarten through grade 12 mathematics instruction. The Standards put forth in that document and addressed in this report are "descriptions of what mathematics instruction should enable students to know and do — statements of what is valued for school mathematics education" (pg. 7). The five Content Standards describe mathematics content goals in the areas of number and operations, algebra, geometry, measurement, and data analysis and probability. The five Process Standards describe goals for problem solving, reasoning and proof, connections, communication, and representation.

In addition to describing goals across the grades, *Principles and Standards* presents the Contents and Process Standards separately for four grade bands: grades pre-K–2, grades 3–5, grades 6–8, and grades 9–12. Within each grade band, "expectations" for what students should know and be able to do are identified.

The NCTM's 1989 *Standards* has been widely recognized as influential in efforts to improve mathematics teaching and learning and it is anticipated that *Principles and Standards* will fulfill the same role. Given the poor to mediocre performance of U.S. students in TIMSS compared with their counterparts in other

countries, achievement should be examined in light of what U.S. mathematics education expects of students. Do the Standards put forth in *Principles and Standards* embody "world-class" achievement as defined by performance on TIMSS? Does *Principles and Standards* include the mathematics understandings and skills that the high-performing students around the world have? Are the concepts and skills with which U.S. students have difficulty articulated clearly in *Principles and Standards?*

To analyze U.S. achievement in light of current priorities for mathematics instruction and students' learning, U.S. performance at the four TIMSS international benchmarks for fourth and eighth grades was compared with the expectations put forth in *Principles and Standards for School Mathematics*. The five mathematics educators from the United States — Lillie Albert, John Dossey, William Hopkins, Chancey Jones, and Mary Lindquist — who had served on the panel that developed the TIMSS international benchmarks, convened again for a three-day meeting. They worked through the *Principles and Standards* and gauged how performance at the TIMSS international benchmarks is reflected in the NCTM Content Standards. For each TIMSS item, the panel determined the Content Standard and expectation for student learning with which aligned. When doing so, the panel considered what students needed to know or be able to do to answer the question or problem correctly and the benchmark to which the item belonged (that is, at which benchmark students were likely to answer the item correctly). With all of the TIMSS items matched with the Content Standards and expectations, the panel evaluated the overall match between the TIMSS tests and *Principles and Standards.*

The comparison shows a solid match between *Principles and Standards* and achievement on TIMSS. In fact, the panel concluded that for the most part, *Principles and Standards* is more rigorous than TIMSS in terms of what is outlined for mathematics teaching. Of the mathematics understandings addressed by TIMSS and reflected in students' performance, very few are missing from *Principles and Standards.* Moreover, *Principles and Standards* includes more topics than are included in the TIMSS tests.

For the most part, the content of the TIMSS fourth-grade test matched the Standards for grades 3–5, and that of the TIMSS eighth-grade test matched the Standards for grades 6–8. Nearly all of the TIMSS fourth-grade items aligned with expectations for grades 3–5, although a few fourth-grade items aligned with expectations for grades Pre-K–2 and a few aligned with expectations for grades 6–8. In other words, the TIMSS fourth-grade items are largely addressing topics that are included in expectations for mathematics instruction in grades 3–5, or earlier. The eighth-grade TIMSS items also had a good match, with all items aligning

with grades 6–8 expectations or with expectations for earlier grades; 28 items aligned with grades 3–5 expectations, one aligned with a grades Pre-K–2 expectation, and no item aligned with a grades 9–12 expectation.

In the next two sections of the report, achievement at the TIMSS international benchmarks is described in more detail through longer descriptions of performance at each level and example items that are typical of what students reaching each benchmark can do. Expectations for mathematics learning in *Principles and Standards* are discussed along with the results for the United States.

As the in-depth examination of U.S. performance shows, much work is needed for U.S. students to meet the world-class standards of performance defined by TIMSS. It is disappointing that few U.S. students reached the higher TIMSS international benchmarks, while in high-performing countries such as Singapore, Korea, Japan, Hong Kong, and the Czech Republic, many students reached the Top 10% and Upper Quarter international benchmarks. Given that for the most part the content of the TIMSS mathematics tests matches the expectations in *Principles and Standards*, one could expect U.S. students to perform well on TIMSS. The discordance between expectations and performance may be because what is articulated in *Principles and Standards* is not being successfully implemented in U.S. classrooms.

A Word about Timing

The TIMSS test was administered while the 1989 *Standards* was in use and before *Principles and Standards* was released in April 2000. Thus, any content, topics, or processes included only in *Principles and Standards* would not necessarily be reflected in the performance of U.S. students in 1995. In comparing *Principles and Standards* and performance on TIMSS, the panel considered any changes between the NCTM's 1989 and 2000 documents. However, there is considerable overlap between the two in terms of the content knowledge and understandings outlined for mathematics education. The major difference is in how the goals for mathematics teaching are communicated in the documents and in the examples given to help teachers implement the Standards.

Fourth Grade

International Benchmarks of Mathematics Achievement

Fourth Grade

Fourth Grade

International
Benchmarks of Mathematics Achievement — Fourth Grade

This section describes student performance at each of the four international benchmarks for fourth grade and shows examples from the sets of items used to describe performance at each benchmark. For each benchmark, from 7 to 9 items are presented. These are items students reaching each benchmark were likely to answer correctly and they represent the kinds of items on which these students are typically successful. For each item, the percent correct for the United States and the highest-performing country on that item are shown, as is the international average percent correct. Because some items will be used in future TIMSS assessments, not all of the items used to develop descriptions of performance are available for display. However, brief descriptions of every item used to develop the benchmark descriptions are provided in the appendix.

FIGURE 3 Grade 4

Top 10% International Benchmark — Fourth Grade

Add/subtract decimal numbers in word problems; convert fractions to decimal numbers; understand verbal and pictorial representations of decimals; understand place value to first decimal place; solve two-step problems involving multiplication and division (one-digit); use simple proportional reasoning to solve problems; relate units of measurement; determine the relationship, involving division, between pairs of whole numbers.

Students at the Top 10% benchmark demonstrate a developing understanding of fractions and decimals and the relationship between them. They can convert fractions with denominators of 10 to decimal numbers , can identify the verbal representation of a decimal number, and can identify the decimal representation of a shaded portion of a figure. They can solve word problems involving addition and subtraction of decimal numbers to one decimal place. They have extended their knowledge of place value to the first decimal place. They can recognize different pictorial representations of the same fraction and can compare and represent two familiar fractions. Students at this level can solve two-step problems involving multiplication and division by one-digit numbers. They are able to find the difference between two products with a common multiplier (such as 25 x 18 and 24 x 18).

Students at this level are able to use simple proportional reasoning to solve word problems involving halves.

Students are familiar with a greater range of metric units, including milliliters and millimeters. They can select an expression for the perimeter of a rectangle given the lengths of two sides. They recognize the inverse relationship between the size of a unit and the number of units required to measure the length of an object. Given a drawing of a three-dimensional geometric figure, students can recognize that figure when it is rotated to a different orientation. They can use a given scale to estimate distance on a map.

Students can select a verbal rule involving division that describes a relationship between pairs of whole numbers given in a table.

Fourth-Grade Achievement at the Top 10% International Benchmark

Figure 3 presents the descriptions of performance at the Top 10% international benchmark for fourth grade. Students reaching this benchmark have demonstrated nearly full mastery of the content of the TIMSS fourth-grade mathematics test. They typically demonstrate success on the knowledge and skills represented by this benchmark, as well as the Upper Quarter, Median, and Lower Quarter benchmarks. Nine percent of U.S. fourth-grade students scored at or above the Top 10% international benchmark, similar to the international percentage. Performance on the example items shown below reflects their performance at this benchmark.

Understanding and Computing with Decimals and Fractions

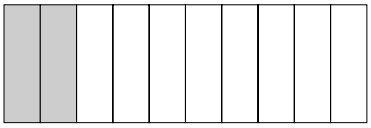
A developing understanding of decimals and fractions is a hallmark of the Top 10% international benchmark at fourth grade. At this level, students have moved beyond an understanding and use of whole numbers typical of lower benchmarks to understanding different representations of fractions (decimal, words, pictorial), being able to convert between different representations of fractions, and doing computation with decimal numbers.

Principles and Standards puts forth expectations for students' understanding and use of fractions and decimals, mainly beginning with the Standards for grades 3–5. Within the Number and Operations Standard, "Understand numbers, ways of representing numbers, relationships among numbers, and number systems," students in grades 3–5 are expected to:

- understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals;
- recognize equivalent representations for the same number and generate them by decomposing and composing numbers;
- develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers;
- use models, benchmarks, and equivalent forms to judge the size of fractions; and
- recognize and generate equivalent forms of commonly used fractions, decimals, and percents.

Example 1 Grade 4

Identifies decimal represen-tation for shaded portion of a rectangular figure divided into 10 equal parts.



Which number represents the shaded part of the figure?

A. 2.8

B. 0.5

C. 0.2

D. 0.02

M05

Percent Correct

★ United States
32

▲ Singapore
81

● International Avg.
40

Although some TIMSS items pertaining to these expectations are typical of performance at lower benchmark levels, it is at the Top 10% benchmark that fourth-grade students demonstrate a better understanding of these concepts. Example items 1, 2, and 3 illustrate students' understanding and use of fractions and decimals.

Example item 1 requires students to recognize that 0.2 represented the shaded part of the figure. On average, 40 percent of fourth-grade students internationally answered the item correctly. In Singapore, 81 percent of fourth-grade students answered this item correctly, while in the United States only 32 percent did so.

Example item 2 is a word problem involving subtraction of decimals. To receive full credit, students had to provide 63.2 as the response and show their calculation of 96.4 – 33.2, or its equivalent. Singaporean fourth graders performed best, with 61 percent answering correctly. Students in the United States performed better than the international average of 26 percent correct, with 32 percent of fourth graders answering correctly.

Example 2 Grade 4

Solves a word problem involving subtraction of decimals to one decimal place (tenths).

Julie put a box on a shelf that is 96.4 centimeters long. The box is 33.2 centimeters long. What is the longest box she could put on the rest of the shelf? Show all your work.

Answer: 63.2 cm. box

96.4
-33.2
63.2

S03

Percent Correct

★ United States
32

▲ Singapore
61

● International Avg.
26

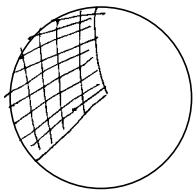
Grade 4 Example 3

Compares two unit fractions and shades part of a circle to justify the answer.

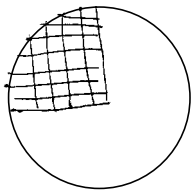
Sam said that $\frac{1}{3}$ of a pie is less than $\frac{1}{4}$ of the same pie.

Is Sam correct? NO

Use the circles below to show why this is so.



Shade in $\frac{1}{3}$ of this circle



Shade in $\frac{1}{4}$ of this circle

V01

Percent Correct

★ **United States** 27

▲ **Singapore** 58

● **International Avg.** 26

In example item 3, students compare two fractions, determine whether 1/3 is less than 1/4, and shade in the circles to justify their answer. Understanding and representing 1/3 and 1/4 is an expectation for grades pre-K–2 students in the NCTM Number and Operations Standard. At grades 3–5 students are expected to "use models, benchmarks, and equivalent forms to judge the size of fractions."

The performance of fourth-grade students in the United States (27 percent correct) was nearly the same as the international average. Internationally, 26 percent of fourth-grade students received full credit (partial credit was awarded to students who could either identify 1/3 as not less than 1/4 or correctly represent the two fractions), while in Singapore, 58 percent answered the item correctly.

Multiplication and Division in Two-Step Word Problems

Within the Number and Operations Standard, the NCTM calls for instructional programs to enable all students to "understand meanings of operations and how they relate to one another." Students in grades 3–5 are expected to "understand various meanings of multiplication and division." Fourth-grade students reaching the lower TIMSS international benchmarks demonstrated that they could multiply and divide whole numbers and understand the meaning of these operations. However, at the Top 10% benchmark, students further extended their understandin g and could do these operations in problems requiring more than one step.

Example 4 Grade 4

Solves a two-step word problem involving division and multiplication by one-digit numbers.

There are 54 marbles, and they are put into 6 bags, so that the same number of marbles is in each bag. How many marbles would 2 bags contain?

A. 108 marbles
B. 18 marbles
C. 15 marbles
D. 12 marbles
E. 9 marbles

K09

Percent Correct

★ United States
36

▲ Singapore
76

● International Avg.
37

In example item 4, students solve a two-step word problem involving multiplication and division, an emerging skill of students reaching this benchmark. Internationally, 37 percent of fourth graders answered this item correctly, and in the United States 36 percent did so. In Singapore, however, about three-quarters (76 percent) of the students were successful on this question.

Simple Proportional Reasoning and Word Problems

Using simple proportional reasoning to solve word problems involving halves is a hallmark of the Top 10% benchmark. In example item 5, students show that they understand that if 5 tomatoes make a half a liter of tomato sauce, then 15 tomatoes would make three times that amount (a liter and a half). Just over half (53 percent) of fourth-grade students internationally selected the correct answer. In the United States, 43 percent did so, while in Hong Kong nearly three-quarters (73 percent) did.

Grade 4 Example 5

Uses proportional reasoning to solve a word problem involving halves.

Mario uses 5 tomatoes to make half a liter of tomato sauce. How much sauce can he make from 15 tomatoes?

(A.) A liter and a half

B. Two liters

C. Two liters and a half

D. Three liters

I05

Percent Correct

★ United States
43

▲ Hong Kong
73

● International Avg.
53

In example item 6, students show pictorially, verbally, or symbolically that a ratio of 10:20 is equivalent to 1:2. This problem was more difficult than example item 5, with only 21 percent of fourth-grade students internationally giving the fully correct response. In the United States, 25 percent successfully completed the task, and in Korea, the highest-performing country on this item, 43 percent of fourth-grade students did so.

Solving problems like examples 5 and 6 involves proportional thinking. Although understanding multiplication relations is expected of grades 3–5 students, it is possible that more emphasis could be placed on the various meanings of multiplication and division, especially those that lead to more formal work with proportions.

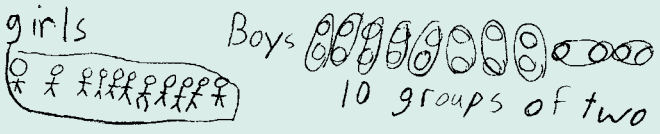
Grade 4 Example 6

Shows that a ratio of 10:20 is equivalent to 1:2 using words or pictures.

There are 10 girls and 20 boys in Juanita's class. Juanita said that there is one girl for every two boys. Her friend Amanda said that means $\frac{1}{2}$ of all the students in the class are girls.

How many students are there in Juanita's class. Answer: 30

Is Juanita right? Answer: yes
Use words or pictures to explain why.



Is Amanda right? Answer: no
Use words and pictures to explain why.

because $\frac{1}{2}$ is 15 not 10 girls and it is not 20 boys

T04A

Percent Correct

★ United States
25

▲ Korea
43

● International Avg.
21

Example 7 Grade 4

States the number of millimeters in a meter.

How many millimeters are in a meter?

Answer: 1000

V05

Percent Correct

★ United States
21

▲ Hong Kong
72

▲ Czech Republic
72

● International Avg.
49

Range of Units of Measurement

Familiarity with a range of metric units is a significant understanding at this benchmark. Within the Measurement Standard ("Understand measurable attributes of objects and the units, systems, and processes of measurement"), grades 3–5 students are expected to "carry out simple unit conversions, such as centimeters to meters, within a system of measurement." However, only 21 percent of U.S. fourth graders knew how many millimeters are in a meter (example item 7). In contrast, nearly half (49 percent) of fourth-grade students internationally knew this, and 72 percent of fourth graders in Hong Kong and the Czech Republic answered this correctly.

In example item 8, students answering the item correctly recognize the inverse relationship between the size of a unit and the number of units required to measure the length of an object. This item was very difficult for fourth graders in the United States. Only 10 percent of U.S. fourth-grade students identified Carlos as the child with the longest pace. Internationally, 32 percent of fourth-grade students could do this, and in Korea 65 percent of fourth graders did.

Example 8 Grade 4

Recognizes the inverse relationship between size of unit and number of units required to cover a distance.

Four children measured the width of a room by counting how many paces it took them to cross it. The chart shows their measurements.

Name	Number of Paces
Stephen	10
Erlane	8
Ana	9
Carlos	7

Who had the longest pace?

A. Stephen

B. Erlane

C. Ana

(D.) Carlos

L08

Percent Correct

★ United States
10

▲ Korea
65

● International Avg.
32

Verbal Rules

Most of the TIMSS algebra items in the fourth-grade test were answered successfully by students performing below the Top 10% benchmark. However, it is not until the Top 10% benchmark that students demonstrate that they can select a verbal rule involving division that describes a relationship between pairs of whole numbers. In example item 9, students figure out how one gets the numbers in column B of the table from the numbers in column A. Although this item aligns with the NCTM expectation (Algebra Standard) for grades 3–5 students to "represent and analyze patterns and functions using words, tables and graphs," in the United States only 32 percent answered the item correctly, below the international average of 39 percent. Fourth-grade students in Korea performed well above those in all other countries, with 70 percent selecting division by 5 as the rule.

Grade 4 Example 9

Identifies the number rule involving division that describes the relationship between pairs of whole numbers given in a table.

What do you have to do to each number in Column A to get the number next to it in Column B?

Column A	Column B
10	2
15	3
25	5
50	10

A. Add 8 to the number in Column A.

B. Subtract 8 from the number in Column A.

C. Multiply the number in Column A by 5.

(D.) Divide the number in Column A by 5.

J05

Percent Correct

★ United States
32

▲ Korea
70

● International Avg.
39

FIGURE 4 Grade 4

Upper Quarter International Benchmark — Fourth Grade

Solve multi-step word problems involving addition, subtraction, and multiplication of whole numbers; solve simple rate and ratio problems; use understanding of place value to solve problems; use information in tables and graphs to solve problems; locate a point on a grid; identify numerical expressions and rules involving multiplication.

Students at the Upper Quarter benchmark can solve multi-step word problems involving addition, subtraction, and multiplication of whole numbers. They can solve simple rate and ratio word problems including those involving 1/2 and 1/4. Students at this level use their understanding of place value to solve problems involving whole numbers.

Students can solve one-step problems involving change in time or temperature. They can select expressions which give the best estimates for the answers to one-step word problems. Students can use given non-standard units to measure length and area.

Students demonstrate familiarity with using information in tables and graphs to solve problems. When reading bar graphs they can determine values associated with unlabeled tick marks. They can use information from tables to solve problems involving addition and subtraction. Given information in a table, they can complete a bar graph by drawing and labeling a set of double bars. Students can locate a point on a rectangular grid, given coordinates or directions.

Students have an intuitive understanding of the likelihood of an event. They can find the next number in a decreasing arithmetic sequence. Students can select numerical expressions that represent verbal expressions and rules involving multiplication.

Achievement at the Upper Quarter International Benchmark

Figure 4 describes performance at the Upper Quarter international benchmark for fourth grade. Students reaching this benchmark typically demonstrate success on the knowledge and skills represented by this benchmark, as well as the Median and Lower Quarter benchmarks.

Twenty-six percent of U.S. fourth-grade students scored at or above the Upper Quarter international benchmark, similar to the international percentage. There were some concepts and skills indicative of this benchmark on which U.S. students performed well, particularly estimation problems and problems requiring students to read and interpret graphs.

Place Value for Whole Numbers

Understanding place value for whole numbers is a hallmark of the TIMSS Upper Quarter international benchmark for fourth grade. The NCTM Number and Operations Standard calls for grades 3–5 students to "understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers…"

Grade 4 Example 10

Orders four digits to make the smallest possible four-digit whole number.

What is the smallest whole number that you can make using the digits 4, 3, 9 and 1 ? Use each digit only once.

Answer: 1349

T02

Percent Correct

★ United States
50

▲ Korea
92

● International Avg.
43

In example item 10, students show they know that 1349 is the smallest number that can be made from those four digits. Half of fourth graders in the United States provided the correct answer, slightly fewer (43 percent) did so internationally, and nearly all (92 percent) of fourth graders in Korea did so.

Example 11 Grade 4

Arranges four digits into two two-digit numbers whose sum is greater than a given number .

In a game, Mysong and Naoki are making addition problems. They each have four cards like these.

1	2	3	4

The winner of the game is the person who can make the problem with the largest answer.

Mysong placed the cards like this.

$$\begin{array}{r} 43 \\ +\ 21 \\ \hline 64 \end{array}$$

Naoki placed the cards like this.

$$\begin{array}{r} 31 \\ +\ 24 \\ \hline 55 \end{array}$$

Who won this game? Mysong

How do you know? 64 is larger than 55

Write numbers in the squares below to show how you would place the cards to beat both Mysong and Naoki.

$$\begin{array}{r} 42 \\ +\ 31 \\ \hline \end{array}$$

(one possible answer)

V04B

Percent Correct

★ **United States** 57

▲ **Hong Kong** 82

● **International Avg.** 48

In example item 11, students arrange four digits into two two-digit numbers whose sum is greater than a given number. They need to understand that the larger of the digits should be put in the tens place. Hong Kong was the top-performing country on the item, with 82 percent of fourth-grade students answering correctly. In the United States, 57 percent did so and internationally 48 percent of fourth graders provided a number large enough to beat Mysong and Naoki.

Multi-Step Word Problems with Whole Numbers

Students reaching the Upper Quarter benchmark demonstrate that they can solve multi-step word problems with whole numbers, going beyond what students at the Median Benchmark can do (solve problems with whole numbers, see below). Example 12 (second part) requires students to figure out how long it will take Louisa to get to school, given the distance and the rate at which she travels. U.S. fourth graders (44 percent correct) performed close to the international average (45 percent correct), but not as well as Hong Kong (70 percent correct).

In example item 13, students solve a two-step word problem involving subtraction. In Korea, 82 percent of fourth-grade students identified 579 as the number that, when subtracted from 900, is greater than 300, while in the U.S. and internationally, less than 60 percent did so.

Grade 4 Example 12

Solves one-step multiplication problem involving rate.

Maria and her sister Louisa leave home at the same time and ride their bicycles to school 9 kilometers away.

Maria rides at a rate of 3 kilometers in 10 minutes. How long will it take her to get to school?

Answer: 30 minutes

Louisa rides at a rate of 1 kilometer in 3 minutes. How long will it take her to get to school?

Answer: 27 minutes

Who arrives at school first?

Answer: Louisa

U03B

Percent Correct

★ United States 44

▲ Hong Kong 70

● International Avg. 45

Grade 4 Example 13

Solves two-step word problem involving subraction.

When you subtract one of the numbers below from 900, the answer is greater than 300. Which number is it?

A. 823

B. 712

C. 667

(D.) 579

I03

Percent Correct

★ United States 57

▲ Korea 82

● International Avg. 57

Example 14 Grade 4

Solves a multi-step problem involving a rate in half-hour time intervals.

A teacher marks 10 of her pupils' tests every half hour. It takes her one and one-half hours to mark all her pupils' tests. How many pupils are in her class?

Answer: 30

S04

Percent Correct

★ **United States** 45

▲ **Netherlands** 73

● **International Avg.** 46

Rate and Ratio Word Problems

At the Upper Quarter benchmark, students demonstrate that they can solve simple rate and ratio word problems, including those involving 1/2 and 1/4. For example, in example item 14, students show that if the teacher can mark 10 tests every half hour and it takes her one and one-half hours to mark all students' papers, she has 30 students. The Netherlands had the highest performance on this item, with 73 percent of fourth graders giving the correct response. The United States (45 percent) performed close to the international average (46 percent). This problem aligns with the expectation (Number and Operations Standard) that grades 3–5 students "understand various meanings of multiplication and division." In this example, students use multiplication in a rate problem to determine the number of students in the teacher's class.

Example 15 Grade 4

Identifies the expression that is the best estimate adding three two-digit numbers.

Elena worked 57 hours in March, 62 hours in April, and 59 hours in May. Which of these is the BEST estimate of the total number of hours she worked for the three months?

A. 50 + 50 + 50

B. 55 + 55 + 55

(C.) 60 + 60 + 60

D. 65 + 65 + 65

J08

Percent Correct

★ **United States** 71

▲ **Singapore** 80

● **International Avg.** 52

Estimation

Students reaching the Upper Quarter benchmark demonstrate understanding of estimation by selecting expressions that give the best estimates for the answers to one-step word problems. In example item 15, students identify 60+60+60 as the best way to estimate the number of hours Elena worked. The United States was among the five top-performing countries on this item, with 71 percent of fourth-grade students responding correctly. Singapore was the highest-performing country (80 percent correct), and internationally about half (52 percent) of fourth graders answered the problem correctly.

The performance of U.S. students on estimation items of this type perhaps reflects the emphasis on fluency with estimation in *Principles and Standards*. For all grades, the Number and Operations Standard calls for students to "compute fluently and make reasonable estimates." At grades 3–5, students are expected to "develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results."

Non-Standard Units of Measurement

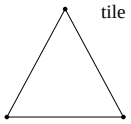
While much of the measurement content in the TIMSS fourth-grade test is mastered by students reaching the Median benchmark (see below), it is not until the Upper Quarter benchmark that students demonstrate they can use given non-standard units to measure length and area.

For example, in item 16, students find the number of triangular tiles it will take to cover the region marked by the dots and show how they worked out their answer. In the United States, 46 percent of fourth-grade students could do this task, just below the international average (50 percent). In Japan, more than three-quarters (77 percent) did so. It may be surprising that U.S. fourth graders did not perform better on this item, since the expectation to "understand how to measure using nonstandard and standard units" appears in the NCTM Measurement Standard for grades pre-K–2.

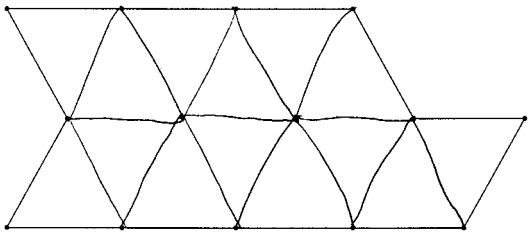
Grade 4 Example 16

Finds the number of triangular tiles needed to cover a region.

The triangle represents one tile in the shape of a triangle.



How many tiles will it take to cover the figure below?



Number of tiles: 14

Use the figure above to show how you worked out your answer.

U01

Percent Correct

★ United States
46

▲ Japan
77

● International Avg.
50

Data Representation and Solving Problems

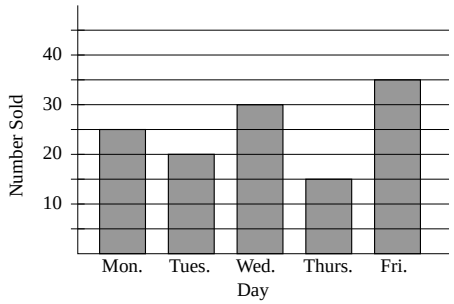
At the Upper Quarter benchmark, students demonstrate familiarity with using information in tables and graphs to solve problems. This is an area in which U.S. fourth graders performed relatively well and that is well covered in *Principles and Standards*. Students in grades pre-K–2 are expected to "represent data using concrete objects, pictures, and graphs." Students in grades 3–5 are expected to "represent data using tables and graphs such as line plots, bar graphs, and line graphs." For grades 6–8 and grades 9–12, *Principles and Standards* calls for students to do more advanced data representation.

In example item 17, students are asked to determine the number of cartons of milk sold in a week, reading a bar graph with unlabeled tick marks and summing across the five days. The United States was the third top-performing country, with 57 percent of fourth graders answering correctly, and performed well above the international average (37 percent correct). Singapore had the highest performance (80 percent correct).

Example 17 Grade 4

Reads data from labeled and unlabeled points on a bar graph to find the total number of objects represented.

The graph shows the number of cartons of milk sold each day of a week at a school.



How many cartons of milk did the school sell on Monday?

Answer: 25

How many cartons of milk did the school sell that week?
Show your work.

25
20
30
15
+ 35
125

Answer: 125

T01B

Percent Correct

★ **United States** 57

▲ **Singapore** 80

● **International Avg.** 37

Example item 18 requires students to use the data provided in the table to complete the bar graph by drawing and labeling a set of double bars. Again, the United States (55 percent correct) performed above the international average (41 percent correct) and was one of the five top-performing countries on this task.

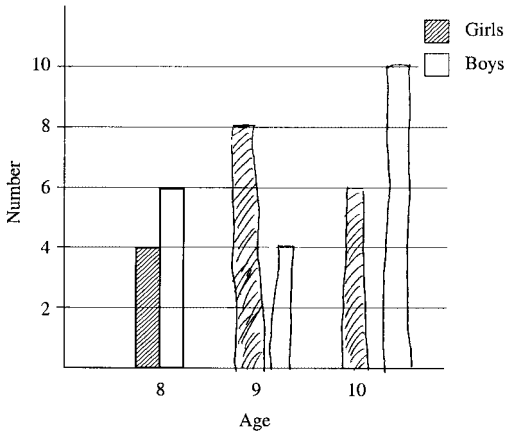
Grade 4 Example 18

Uses numerical data from a table to draw sets of double bars to complete a graph.

This table shows the ages of the girls and boys in a club.

Age	Number of Girls	Number of Boys
8	4	6
9	8	4
10	6	10

Use the information in the table to complete the graph for ages 9 and 10.



S01

Percent Correct

★ United States
55

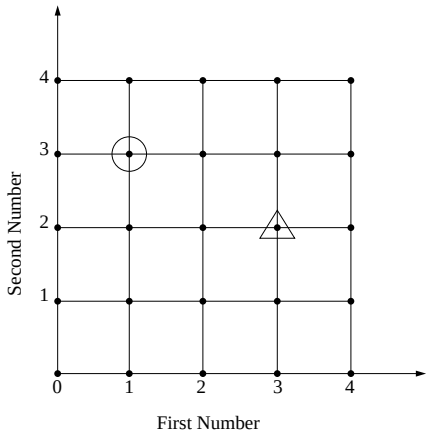
▲ Korea
83

● International Avg.
41

Example 19 Grade 4

Given the coordinates of a dot on a grid, determines the coordinates of another dot on the grid.

On this grid, find the dot with the circle around it. We can describe where this dot is by saying it is at First Number 1, Second Number 3



Now find the dot with the triangle around it. Describe where the dot is on the grid in the same way. Fill in the numbers we would use:

First number 3 Second Number 2

M04

Percent Correct

★ **United States** 56

▲ **England** 68

● **International Avg.** 42

Coordinates

In geometry, students reaching the Upper Quarter benchmark show they can locate a point on a rectangular grid, given coordinates or directions. This aligns with the NCTM expectation (Geometry Standard) for students in grades 3–5 to "make and use coordinate systems to specify locations and to describe paths."

In example item 19, students are asked to find the numbers (coordinates) identifying the location of the dot with the triangle around it. The United States (56 percent correct) performed below the top-performing country, England (68 percent correct), yet above the international average (42 percent correct).

Numerical Expressions

Students reaching the Upper Quarter benchmark demonstrate that they can select numerical expressions that represent verbal expressions and rules involving multiplication. At the Median benchmark (see below) students find equivalent expressions involving addition and subtraction; doing so with multiplication represents increased understanding.

Grade 4 Example 20

Selects the number pairs satisfying a specified number operation rule involving multiplication.

Which pair of numbers follows the rule "Multiply the first number by 5 to get the second number"?

A. 15 → 3

B. 6 → 11

C. 11 → 6

(D.) 3 → 15

K03

Percent Correct

★ **United States** 65

▲ **Korea** 80

● **International Avg.** 53

In example item 20, students identify 3 ⟶ 15 as representing "multiply the first number by 5 to get the second number." Internationally, just over half (53 percent) of fourth-grade students identified the correct expression. The United States performed above the international average, with 65 percent correct, but not as well as top-performing Korea, which had 80 percent correct. This item aligns with the NCTM Algebra Standard for grades 3–5 that calls for students to "represent and analyze patterns and functions using words…."

FIGURE 5 Grade 4

Median International Benchmark — Fourth Grade

Add, subtract, and multiply whole numbers with regrouping; recognize familiar fractions; add/subtract decimals; demonstrate familiarity with units of mass; read, locate, and compare data in tables and graphs; recognize and use lines of symmetry; recognize and complete number sentences with addition, subtraction, multiplication.

Students at the Median Benchmark can solve one-step word problems involving addition and subtraction of whole numbers and similar problems involving multiplication by a one-digit whole number with regrouping. They can subtract whole numbers involving multiple regroupings and zeroes. Students demonstrate knowledge of place value to the thousandths in situations involving expanded notation of four- and five-digit whole numbers. They can round whole numbers to the nearest hundred. They recognize, for small numbers, that multiplication is commutative. Students demonstrate some familiarity with fractions and decimals. They can add and subtract decimals involving hundredths and regrouping. They recognize a shaded region representing a familiar fraction and can write a fraction greater than a given fraction with a nominator and denominator smaller than 10.

Students know about and can work with units of mass (weight). For example, they can select an appropriate metric unit to measure mass (weight), compare metric units of mass (weight), and select an appropriate value for the mass (weight) of a familiar object. Students can read, locate, and compare data in tables and graphs and can interpret symbols used in pictographs. They can count forward in weeks on a calendar.

Students at this level recognize lines of symmetry and can use a line of symmetry to draw a complete figure when half of the figure is presented. They recognize flat and curved surfaces of a solid such as a cylinder. Given a set of overlapping geometric shapes, students can locate a point in a region described in a statement involving the logical connectives "and" and "not".

Students can identify a whole number that satisfies an inequality involving multiplication. They can recognize a number sentence that represents a word problem involving addition or subtraction. Given two "greater than" relationships involving three quantities, students can determine the third "greater than" relationship. Students can find a specified term in a sequence given the first three terms pictorially.

Achievement at the Median International Benchmark

Figure 5 describes performance at the Median benchmark for fourth grade. Students reaching this benchmark typically demonstrate success on the knowledge and skills represented by this benchmark, as well as the Lower Quarter benchmark. Fifty-six percent of U.S. fourth-grade students scored at or above this benchmark, similar to the international percentage.

Whole Numbers, Fractions, and Operations

The Median benchmark for fourth grade is characterized by proficiency in using whole numbers and a beginning understanding of fractions and decimals. The NCTM Number and Operation Standard emphasizes understanding and representing whole numbers for grades pre-K–2. By grades 3–5, students are expected to apply that understanding to solve problems and to extend their understanding of numbers, ways of representing numbers, and relationships among numbers to fractions. U.S. fourth-grade students perform at or near the international average on the items typical of the performance at this benchmark.

As illustrated by items 21 and 22, students reaching the Median benchmark can add and subtract whole numbers of up to four digits and can multiply one-digit whole numbers. The United States performed at or just under the international average on both items. At least 90 percent of students in Korea and Singapore answered these two items correctly.

Grade 4 Example 21

Subtracts one four-digit number from another in a problem involving multiple regrouping and zeros.

Subtract: 6000 − 2369

A. 4369
B. 3742
C. 3631
D. 3531

I09

Percent Correct

★ United States 71

▲ Korea 93

▲ Singapore 92

● International Avg. 71

Grade 4 Example 22

Provides missing term in expanded notation of four-digit number.

Here is a number sentence.

2000 + ☐ + 30 + 9 = 2739

What number goes where the ☐ is to make this sentence true?

Answer: 700

S02

Percent Correct

★ United States 58

▲ Singapore 92

▲ Korea 91

● International Avg. 63

Example 23 Grade 4

Writes a fraction larger than a given fraction.

Write a fraction that is larger than $\frac{2}{7}$.

(One Possible Answer)

Answer: 3/7

U02

Percent Correct

★ United States
68

▲ Korea
90

● International Avg.
57

In example item 23 students write a fraction that is larger than 2/7, illustrating their beginning understanding of fractions. Ninety percent of fourth-grade students in Korea supplied a fraction larger than 2/7. Sixty-eight percent of U.S. fourth graders did so, performing above the international average of 57 percent correct.

Example 24 Grade 4

Recognizes that the order of two numbers in multiplication problem involving two numbers does not affect the answer.

☐ stands for a number. 7 × ☐ will always give the same answer as

(A.) ☐ × 7
B. ☐ + 7
C. ☐ – 7
D. 7 + ☐
E. ☐ ÷ 7

M03

Percent Correct

★ United States
71

▲ Japan
89

● International Avg.
63

Item 24 illustrates students' understanding that multiplication is commutative. Internationally, 63 percent of fourth graders selected the term that would provide the same answer as that given in the stem. The United States performed above the international average with 71 percent correct; in Japan, 89 percent of students answered correctly. This item aligns with the NCTM Number and Operations Standard calling for students in grades 3–5 to "understand various meanings of multiplication…."

Units of Mass

A significant accomplishment of students teaching the Median benchmark is knowing about and being able to work with units of mass, such as selecting an appropriate metric unit to measure mass and comparing metric units of mass. Students in grades 3–5 are expected to "understand such attributes such as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute", and "understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems."

Although *Principles and Standards* calls for students in grades 3–5 to understand and use metric units, U.S. fourth graders did not perform as well as their counterparts in other countries on example item 25, in which students compare metric units of mass by identifying a kilogram as the largest of four such units. U.S. fourth graders (61 percent correct) performed below the international average (72 percent correct). Nearly all students in Japan (94 percent) selected the appropriate unit.

Grade 4 Example 25

Selects the largest metric unit of weight (mass).

Which of these is largest?

(A.) 1 kilogram

B. 1 centigram

C. 1 milligram

D. 1 gram

J06

Percent Correct

★ United States
61

▲ Japan
94

● International Avg.
72

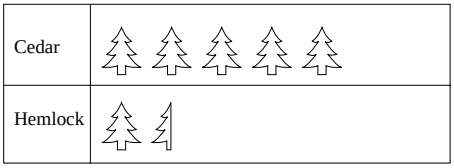
Reading Data in Tables and Graphs

Students at the Median benchmark have the table and graph reading skills that are a prerequisite for the more advanced problem solving with data in tables and graphs typical of performance at the Upper Quarter benchmark. At the Median benchmark, students read, locate, and compare data in tables and graphs, and interpret symbols used in pictographs. This last skill is illustrated in item 26, in which students interpret the numerical value of a tree, given information about the number of cedar and hemlock trees in a pictograph. Fourth-grade students in the United States (68 percent correct) performed better than the international average (49 percent), but not as well as highest-performing Japan (90 percent correct).

Example 26 Grade 4

Interprets the numerical value of a symbol on a pictograph.

The graph shows 500 cedar trees and 150 hemlock trees.



How many trees does each  represent?

Answer: 100

L01

Percent Correct

★ United States
68

▲ Japan
90

● International Avg.
49

Symmetry

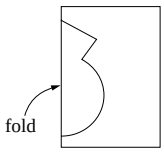
In geometry, students at this level demonstrate some familiarity with shapes, and have a basic understanding of symmetry. This is addressed in the NCTM Geometry Standard, which calls for grades 3–5 students to "identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs."

As illustrated by item 27, students reaching the Median benchmark use a line of symmetry to draw a complete figure when half of the figure is shown. Internationally, 59 percent of fourth graders could do this task and in the United States 66 percent did so. In Japan, 88 percent of students drew the correct shape.

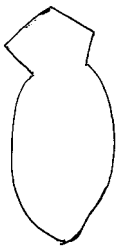
Grade 4 Example 27

Given half of a symmetric figure, draws the complete figure.

Craig folded a piece of paper in half and cut out a shape.



Draw a picture to show what the cut-out shape will look like when it is opened up and flattened out.



T05

Percent Correct

★ United States 66

▲ Japan 88

● International Avg. 59

Example 28 Grade 4

Finds a specified term of a sequence given the first three terms pictorially.

Here is the beginning of a pattern of tiles.

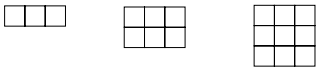


Figure 1 Figure 2 Figure 3

If the pattern continues, how many tiles will be in Figure 6 ?

A. 12

B. 15

C. 18

D. 21

K06

Percent Correct

★ United States
59

▲ Korea
87

● International Avg.
63

Algebra

While at the Lower Quarter benchmark (see below), algebraic knowledge and understanding is limited to recognizing simple patterns, at the Median benchmark students can extend a simple pattern presented pictorially and recognize and complete number sentences with addition, subtraction, and multiplication.

As shown in example item 28, students find the number of tiles in the sixth figure of the pattern of tiles. The correct number of tiles was identified by 63 percent of fourth-grade students internationally, 59 percent in the United States, and 87 percent in highest-performing Korea.

In example item 29, students identify the number sentence that would enable Tanya to determine the number of pages she must read to finish her book. Fourth graders in the United States performed above the international average (71 percent correct compared with 62 percent correct). Eighty-eight percent of students in Japan identified the correct number sentence.

Items 28 and 29 align with expectations within the NCTM Algebra Standard which calls for students in grades 3–5 to "describe, extend, and make generalizations about geometric and numeric patterns" and "represent the idea of a variable as an unknown quantity using a letter or a symbol."

Grade 4 Example 29

Recognizes the number sentence used to represent a word problem involving addition or subtraction.

Tanya has read the first 78 pages in a book that is 130 pages long. Which number sentence could Tanya use to find the number of pages she must read to finish the book?

A. 130 + 78 = ☐

B. ☐ - 78 = 130

C. 130 ÷ 78 = ☐

D. 130 – 78 = ☐

I07

Percent Correct

★ United States
71

▲ Japan
88

● International Avg.
62

FIGURE 6 Grade 4

Lower Quarter International Benchmark — Fourth Grade

Name, order, and use four-digit numbers in a variety of representations; add whole numbers; multiply small whole numbers without regrouping; read and locate information in simple graphs; recognize representations of one-half, triangles, and simple patterns.

Students at the Lower Quarter benchmark demonstrate an understanding of whole numbers. They are familiar with numbers into the thousands. They can translate between representations of whole numbers — expanded forms, pictorial representations, and words. They can add two four-digit numbers involving three regroupings, recognize whether addition or multiplication is the appropriate operation for solving a problem involving whole numbers, and select the correct answer for a multiplication problem involving one-digit and two-digit numbers. Students also grasp the relationship between repeated addition of whole numbers and multiplication. Students can recognize one-half of a set of objects.

In measurement, students at this level can estimate the length of a short object in centimeters and compare the areas of two rectangular regions by counting the equal squares that cover their areas. Students can locate up to two pieces of information in simple bar and circle graphs. They recognize that there is a better chance of landing on a shaded part of an area when a greater proportion of the area is shaded.

Students at this level have some understanding of triangles and straight versus curved sides, recognize figures with the same shape and size, and locate an object on a simple grid. They can recognize simple patterns involving whole numbers and geometric shapes.

Achievement at the Lower Quarter International Benchmark

Figure 6 describes performance at the Lower Quarter benchmark for fourth grade. Students reaching this benchmark typically demonstrate success on the knowledge and skills represented by this benchmark. Eighty-three percent of U.S. fourth-grade students scored at or above the Lower Quarter benchmark, similar to the international percentage.

Understanding and Using Whole Numbers

Performance at the Lower Quarter benchmark is dominated by an understanding and use of whole numbers, with very little demonstration of understanding fractions. In example item 30, students identify the largest of four four-digit numbers. Students add two four-digit numbers in example item 31, which involves three regroupings. In example item 32, students multiply two whole numbers. As illustrated in example item 33, students grasp the relationship between repeated addition of whole numbers and multiplication.

On these four example items, fourth-grade students in the United States performed about as well as or just above the international average, demonstrating a strong grasp of whole numbers. Their performance aligns with the NCTM Number and Operations Standard, which calls for whole numbers to be covered in grades pre-K–2 and grades 3–5.

Grade 4 Example 30

Identifies the largest four-digit number pattern formed by four different, given, digits.

Which of these is the largest number?

A. 2735

B. 2537

C. 2573

D. 2753

M08

Percent Correct

★ United States
89

▲ Korea
97

● International Avg.
86

See next page for Examples 31–34

Example 31 Grade 4

Adds two four-digit numbers involving three regroupings.

Add: 6971
+5291

A. 11 162

B. 12 162

C. 12 262

D. 1 211 162

K02

Percent Correct

★ United States
86

▲ Korea
96

● International Avg.
84

Example 32 Grade 4

Multiplies a two-digit number by a single-digit number with no regrouping.

What is 3 times 23 ?

A. 323

B. 233

C. 69

D. 26

I04

Percent Correct

★ United States
90

▲ Netherlands
98

● International Avg.
84

Example 33 Grade 4

Represents simple repeated addition as multiplication.

Addition Fact
4 + 4 + 4 + 4 + 4 = 20

Write this addition fact as a multiplication fact.

4 × 5 = 20

U05

Percent Correct

★ United States
84

▲ Hong Kong
95

● International Avg.
77

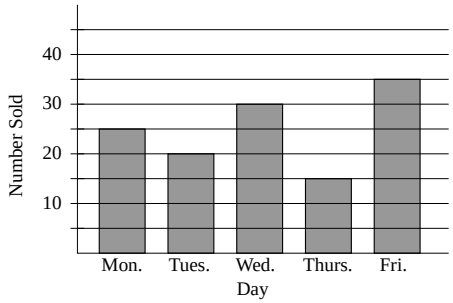
Locating Information in Graphs

Locating information in simple bar and circle graphs is an accomplishment of students at the Lower Quarter benchmark; this is illustrated in example item 34 (first part), in which students read the bar for Monday to find the number of cartons of milk sold by the school on that day. Fourth-grade students in the United States performed very well on this item (90 percent correct), above the international average of 75 percent correct and close to the top-performing countries. The second part of this task, which requires students to find the cartons sold in a week (counting across 5 days), is typical of performance at the Upper Quarter benchmark (see above); students in the United States performed comparatively well on that part of the task as well.

Grade 4 Example 34

Reads an unlabeled scale point on a simple bar graph.

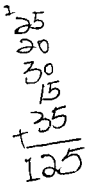
The graph shows the number of cartons of milk sold each day of a week at a school.



How many cartons of milk did the school sell on Monday?

Answer: 25

How many cartons of milk did the school sell that week?
Show your work.



Answer: 125

T01A

Percent Correct

★ **United States** 90

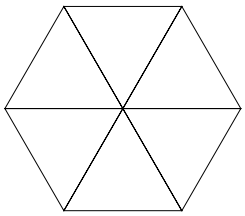
▲ **Korea** 96

● **International Avg.** 75

Example 35 Grade 4

Recognizes the figure Δ is a triangle.

Here is a hexagon.



The hexagon is divided into six

A. triangles
B. squares
C. pentagons
D. rectangles

J01

Percent Correct

★ **United States** 91

▲ **Czech Republic** 97

▲ **Hong Kong** 97

● **International Avg.** 88

Recognize Triangles

In geometry, students at the Lower Quarter benchmark recognize triangles, as illustrated in example item 35. Ninety-one percent of U.S. fourth graders identified the six figures in the hexagon as triangles, close to highest-performing Czech Republic and Hong Kong (both with 97 percent correct).

Recognize Simple Patterns

Students reaching the Lower Quarter benchmark recognize simple patterns involving whole numbers and geometric shapes. In example item 36, students select the pattern of shapes that matches the pattern in the question stem. Only two countries had percents correct on the item above 90 (Korea with 93 percent and Japan with 91 percent); the international average was 73 percent correct. U.S. fourth graders performed slightly better than the international average, with 79 percent correct.

Grade 4 Example 36

Recognizes same pattern of sequence of shapes when made with different shapes.

These shapes are arranged in a pattern.

○△○○△△○○○△△△

Which set of shapes is arranged in the same pattern?

A. ★□★□★★□□★★□□

B. □★□□★□□□★□□□□

C. ★□★★ □□★★★□□□

D. □□★★□★□□★★□★

L04

Percent Correct

★ United States
79

▲ Korea
93

▲ Japan
91

● International Avg.
72

Eighth Grade

International Benchmarks of Mathematics Achievement

Eighth Grade

Eighth Grade

International
Benchmarks of Mathematics Achievement — Eighth Grade

This section describes student performance at each of the four international benchmarks for eighth grade and shows examples from the sets of items used to describe performance at each benchmark. For each benchmark, from 6 to 12 items are presented. These are items that students reaching each benchmark were likely to answer correctly and they represent the kinds of items on which students are successful. For each item the percent correct for the United States and the highest-performing country on that item are shown, as is the international average percent correct. Because some items will be used in future TIMSS assessments, not all of the items used to develop descriptions of performance are available for display. However, brief descriptions of every item used to develop the profiles are provided in the appendix.

FIGURE 7 Grade 8

Top 10% International Benchmark — Eighth Grade

Organize information in problem-solving situations; solve time-distance-rate problems involving conversion of measures within a system; apply relationships – fractions and decimals, ratios, properties of geometric figures, and algebraic rules – to solve problems; solve word problems involving the percentage of increase.

Students at the Top10% benchmark demonstrate the ability to organize information in problem-solving situations. They can select, arrange, and process information from two advertisements to solve a complex word problem involving whole numbers and different units of measurement. They extract and organize information about a part and ratio in a pattern of squares to solve for the total and explain the strategy for estimating the sum of times given in minutes and seconds.

Students use their understanding of fractions and decimals in multi-step problem situations. They can solve word problems involving fractions and decimals which require analysis of the verbal relations described. They can solve a time-distance rate problem involving decimals and conversion of minutes to seconds and, given the total and ratio of parts, determine the value of one part. They can solve word problems involving the percentage of increase.

Students can apply their knowledge of measurement in more complex problem situations. They apply knowledge of properties of squares and the relationship between perimeter and area to solve a multi-step word problem and, given the dimensions of a rectangle, draw a new rectangle whose dimensions are specified fractional multiples of the original dimensions. They understand different units of time and can apply their knowledge of the number of milliliters in a liter to solve a word problem.

In geometry, students can use their knowledge of congruent and similar triangles to solve more complex problems concerning corresponding parts. They can identify the coordinates of a point on a line given the coordinates of two other points on the line.

Students can identify what a variable represents in an equation for a given situation and can identify the equation (formula) corresponding to a verbal description relating two variables. They can identify an equivalent form of a linear inequality involving a fraction.

Given a table of values containing ordered pairs (x,y), students can determine a rule that describes the relationship between the ordered pairs and find the value for a missing term in the table. Students can find two whole numbers given the sum and difference of the two numbers in a word-problem setting.

Students demonstrate emerging algebraic reasoning skills. They can visualize a physical situation presented in a word problem to determine a sequence and find the sum of several terms. They can use algebraic reasoning to determine a sequence of square numbers (perfect squares) and find a specified term.

Achievement at the Top 10% International Benchmark

Figure 7 describes performance at the Top 10% benchmark for eighth grade. Students reaching this benchmark have demonstrated nearly full mastery of the content of the TIMSS eighth-grade mathematics test. They typically exhibit the knowledge and skills represented by this benchmark, as well as the Upper Quarter, Median, and Lower Quarter benchmarks.

Few (5 percent) eighth-grade students in the United States reached the Top 10% benchmark. In contrast, in top-performing Singapore nearly half (45 percent) of eighth-grade students reached this level, and more than a quarter of students did so in Hong Kong, Japan, and Korea.

Organizing and Processing Information to Solve Multi-Step Word Problems

A hallmark of the Top 10% international benchmark for eighth grade is that students solve problems in which they need to organize the information presented and carry through multiple steps. They show they can "bring things together" to solve problems set in a variety of contexts.

Problem solving receives heavy emphasis in *Principles and Standards*. Problem solving is viewed as "an integral part of all mathematics learning" and is included throughout the different Content Standards. In addition, the Problem Solving Process Standard emphasizes the importance of problem solving in all content areas and across all grades. The Standard calls for instructional programs to enable students to

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems; and
- monitor and reflect on the process of mathematical problem solving.

Although problem solving is an important part of NCTM's goals for mathematics instruction, U.S. performance on multi-step problems is typically average to poor, with eighth graders tending to fail to carry through all of the steps required to solve a problem.

Example 1 Grade 8

Extracts information from a complex problem situation and uses a part and a ratio to find the total.

Two boxes of square-shaped cardboard pieces are available to make a larger pattern. There are 4 small squares in each piece.

All pieces in Box 1 look like 

All pieces in Box 2 look like 

In the required pattern, for every piece from Box 2 there are 2 pieces from Box 1.

a) If 60 pieces from Box 2 are used in the required pattern, how many pieces will be needed altogether?

Answer: 180

60
120

b) What fraction of the small squares in the required pattern will be black?

Total = 720
Box 1 = 120
Box 2 = 120
240 black

240/720 = 1/3

Answer: 1/3

T02A

Percent Correct

★ **United States** 15

▲ **Singapore** 47

● **International Avg.** 23

Example item 1 (part a) illustrates students' ability to extract and organize information in a problem situation. Students have to process the information given about the cardboard pieces and the larger pattern to be created from the small pieces. Students use this information and information about one part of the pattern to determine the total number of pieces needed altogether to make the larger pattern. Nearly half (47 percent) of Singaporean eighth-graders successfully solved this problem and determined the correct number of pieced needed (180), while in the United States only 15 percent did so. In the United States, the most common (32 percent) incorrect response was 120, indicating that many students multiplied the number of pieces from Box 2 by 2 to get 120, but failed to go one step further to add that to the number of pieces from Box 1 (60).

As illustrated in example item 2, students reaching the Top 10% benchmark could select the information needed from the two advertisements and organize this information to determine which office building to rent in order to get the lower price. The problem involves different units (cost per month versus cost per square meter per year), and the advertisements include some extraneous information. Students who identified Building A as that with the lower price and showed accurate computations to support this conclusion received full credit. Performance on this problem was quite low in many TIMSS countries (international average of 19 percent correct) including the United States (18 percent correct). Only in Singapore (55 percent correct) and Korea (50 percent) did half or more of eighth-grade students provide a fully correct answer to this problem.

Grade 8 Example 2

Selects, organizes, and processes relevant information from two advertisements to solve word problem invlolving whole numbers and different units.

The following two advertisements appeared in a newspaper in a country where the units of currency are *zeds*.

BUILDING A	**BUILDING B**
Office space available 85 - 95 square meters 475 *zeds* per month 100 - 120 square meters 800 *zeds* per month	Office space available 35 - 260 square meters 90 *zeds* per square meter per year

If a company is interested in renting an office of 110 square meters in that country for a year, at which office building, A or B, should they rent the office in order to get the lower price? Show your work.

Price of Renting 1 m² in Building A = 80 × 12
in a year = 9600 (zeds)

Price of Renting in Building B = 110 × 90
in a year = 9900 (zeds)

∴ 9600 < 9900

∴ They should rent the office at Building A in order to get the lower price.

V02

Percent Correct

★ United States
18

▲ Singapore
55

● International Avg.
19

Example 3 Grade 8

Solves multi-step problem with fractions requiring processing and organizing information.

Jan had a bag of marbles. She gave half of them to James and then a third of the marbles still in the bag to Pat. She then had 6 marbles left. How many marbles were in the bag to start with?

(A.) 18

B. 24

C. 30

D. 36

N16

Percent Correct

★ United States
39

▲ Singapore
66

● International Avg.
47

Example 4 Grade 8

Solves word problem involving the percent of increase and estimation.

Last year there were 1172 students at Beaton High School. This year there are 15 percent more students than last year. Approximately how many students are at Beaton High School this year?

A. 1800

B. 1600

C. 1500

(D.) 1400

E. 1200

K06

Percent Correct

★ United States
40

▲ Singapore
67

● International Avg.
43

Fractions and Percents

The NCTM Number and Operations Standard calls for grades 6–8 students to "work flexibly with fractions, decimals, and percents to solve problems." Example items 3 and 4 are two items that address this expectation on which students reaching the Top 10% benchmark were typically successful. While students reaching lower international benchmarks were successful on some items involving fractions, decimals, and percents, these two examples were likely more difficult because they involved multiple steps.

Item 3 is a multi-step word problem that requires students to sort through and organize the information presented, analyze the relationships presented verbally, and use their understanding of fractions to determine the correct answer. About two-thirds (66 percent) of eighth-grade students in Singapore identified 18 as the number of marbles that Jan had in the bag. In the United States, 39 percent did so, below the international average of 47 percent.

Students reaching the Top 10% benchmark understand how to determine and use the percentage of increase to solve problems, something not demonstrated by many students performing below this benchmark. Item 4 illustrates how students reaching this benchmark used this understanding to determine the number of students at Beaton High School. In the United States, 40 percent of eighth graders correctly determined that there were 1400 students at Beaton High School. Two-thirds (67 percent) of Singaporean eighth graders did so.

Using Measurement in Complex Problems

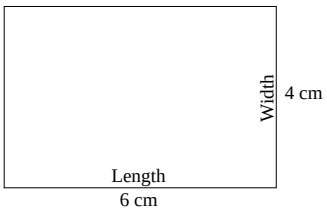
Example Item 5 (part a) illustrates how students reaching the Top 10% benchmark use their knowledge of measurement in complex problem situations. In part of a this problem, students were shown a rectangle with its dimensions in centimeters and asked to draw a new rectangle one and one-half times the length and half the width of the given rectangle and show the dimensions of this new rectangle. Students that correctly drew and labeled a 9 cm by 2 cm rectangle were given full credit. In most countries, students had considerable difficulty with this task. On average, just 31 percent of eighth graders provided a correct drawing of the new rectangles; in the United States, only 16 percent did so. Korea and Austria were the only two countries in which at least half of eighth-grade students completed the task successfully.

Part b of this problem, in which students were asked the ratio of the area of the new rectangle to the area of the first, was even more difficult for students; fewer than half of eighth graders reaching the Top 10% benchmark and only 10 percent of eighth graders overall provided a fully correct response.

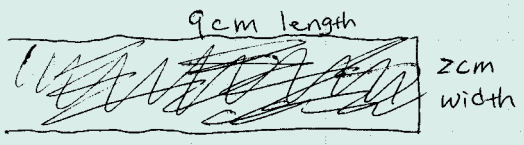
It is disappointing that U.S. eighth graders performed poorly on example item 5, since the NCTM Measurement Standard calls for students in grades 6–8 to "solve problems involving scale factors, using ratio and proportion." However, it is perhaps not surprising that few U.S. eighth graders correctly drew the new rectangle, as they typically did not do well on problems involving processing information and carrying through more than one step. In this case, however, few U.S. students failed to get even partial credit, which they could have obtained by giving the correct dimensions but drawing the rectangle on the grid incorrectly, or by providing the correct length or width, but not both.

Grade 8 Example 5

Given dimensions of a rectangle, draws a new rectangle whose dimensions are specified fractional multiples of the original dimensions.



a. In the space below, draw a new rectangle whose length is one and one half times the length of the rectangle above, and whose width is half the width of the rectangle above. Show the length and width of the new rectangle in centimeters on the figure.



b. What is the ratio of the area of the new rectangle to the area of the first one?

Show your work. 18 : 24 = 3 : 4

U02A

Percent Correct

★ **United States** 16

▲ **Korea** 54

▲ **Austria** 51

● **International Avg.** 31

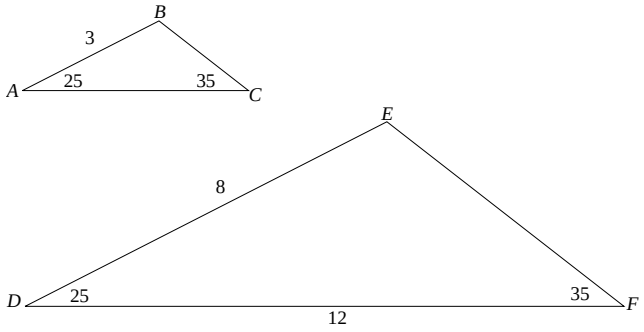
Triangles and Coordinates

Students reaching the Top 10% benchmark have developed an understanding of congruent and similar triangles, and they use this understanding to solve problems. To illustrate, in example item 6 students were shown two similar triangles, given the lengths of two sides of each triangle, and asked to determine the length of the third side of one of the triangles. Students reaching the Top 10% benchmark could identify the correct length. This item was somewhat difficult for U.S. eighth graders (28 percent correct), while in Japan 71 percent of eighth-grade students determined that 4.5 was the correct length of side AC. It is unfortunate that U.S. students found this item difficult, since the NCTM Geometry Standard includes the expectation that grades pre-K–2 students "explore congruence and similarity" and that grades 6–8 students "understand relationships among angles, side lengths, perimeters, areas, and volumes of similar objects."

Example 6 Grade 8

Finds the length of corresponding sides of similar triangles.

Triangles *ABC* and *DEF* are similar triangles.



What is the length of side *AC* ?

A. 2

B. 4

C. 4.5

D. 5.5

E. 32

P09

Percent Correct

★ United States
28

▲ Japan
71

● International Avg.
39

Students reaching the Top 10% benchmark also demonstrate that they can identify coordinates of a point on a line, given two other points. As illustrated by example item 7, when given that a straight line passes through points (3,2) and (4,4), students reaching this level could identify that the point at coordinates (5,6) also lies on the line. Internationally and in the United States, 41 percent of eighth graders correctly answered this item. In the Netherlands, 66 percent of students did so.

Grade 8 Example 7

Given the coordinates of two points on a line, identifies the coordinates of a third point.

A straight line on a graph passes through the points (3,2) and (4,4). Which of these points also lies on the line?

A. (1,1)

B. (2,4)

C. (5,6)

D. (6,3)

E. (6,5)

I08

Percent Correct

★ **United States** 41

▲ **Netherlands** 66

● **International Avg.** 41

Algebra: Inequalities, Rules, and Reasoning

While students reaching the eighth-grade Upper Quarter benchmark (see below) can typically work with linear equalities and expressions, those reaching the Top 10% benchmark can also work with inequalities. For example, in item 8 students identify the equivalent form of a given inequality involving fractions. U.S. eighth graders did relatively well on this item (52 percent correct), although in Singapore and Japan 69 percent of eighth-grade students identified the equivalent form of the inequality.

The NCTM Algebra Standard for grades 6–8 includes the expectation that students recognize and generate equivalent forms for simple algebraic expressions and solve linear equations; equivalent forms of inequalities are included in the Algebra Standard for grades 9–12.

Understanding and applying relationships to solve problems is a hallmark of the Top 10% benchmark, as evidenced by students' understanding of ratios, properties of geometric figures, and numbers. Understanding relationships between numbers is illustrated in example item 9, in which students determine the rule that describes the relationship between ordered pairs of numbers in a table and use the rule to find the missing number in the table. While 60 percent of Korean eighth graders could determine and apply the rule to identify 3 as the missing number, only 39 percent of U.S. eighth graders could do so. This item aligns with the NCTM Algebra Standard for grades 6–8, which calls for students to "relate and compare different forms of representation for a relationship."

Example 8 Grade 8

Given a linear inequality involving fractions, identifies an equivalent form.

$\frac{x}{2} < 7$ is equivalent to

A. $x < \frac{7}{2}$

B. $x < 5$

C. $x < 14$

D. $x > 5$

E. $x > 14$

K04

Percent Correct

★ **United States** 52

▲ **Singapore** 69

▲ **Japan** 69

● **International Avg.** 44

Example 9 Grade 8

Finds and uses rule describing the relationship between ordered pairs given in a table.

The table represents a relation between x and y.

What is the missing number in the table?

A. 2

B. 3

C. 4

D. 5

E. 6

x	y
1	1
2	?
4	7
7	13

J18

Percent Correct

★ **United States** 39

▲ **Korea** 60

● **International Avg.** 41

Students reaching the Top 10% benchmark demonstrate emerging algebraic reasoning skills. In example item 10 (part b), students are shown a sequence of three similar triangles made up of smaller triangles. They determine the pattern of square numbers and the number of triangles that would be needed to create the eighth figure in the sequence.

Although the NCTM Algebra Standard calls for grades 6–8 students to "represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules," this problem was difficult for eighth-grade students in the United States (25 percent correct). U.S. performance was comparable to the international average (26 percent correct), although half or more of eighth graders in Japan and Singapore could extend the pattern and determine that 64 small triangles would be needed for the eighth figure.

Grade 8 Example 10

Uses algebraic reasoning to determine a pattern of square numbers and find a specified term.

Here is a sequence of three similar triangles. All of the small triangles are congruent.

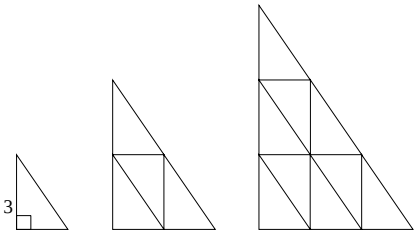
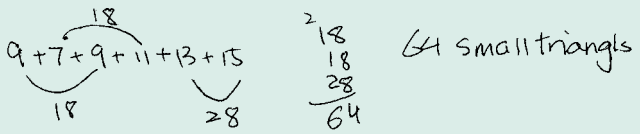


Figure 1 Figure 2 Figure 3

a. Complete the chart by finding how many small triangles make up each figure.

Figure	Number of small triangles
1	1
2	4
3	9

b. The sequence of similar triangles is extended to the 8th Figure. How many small triangles would be needed for Figure 8?



S01B

Percent Correct

★ **United States** 25

▲ **Japan** 52

▲ **Singapore** 50

● **International Avg.** 26

FIGURE 8 Grade 8

Upper Quarter International Benchmark — Eighth Grade

Order, relate, multiply, and divide fractions and decimals; relate area and perimeter; understand simple probability; use knowledge of geometric properties to solve problems; identify algebraic expressions and solve equations with two variables.

Students at the Upper Quarter Benchmark demonstrate an increased facility with fractions and decimals through computation, ordering, rounding and use in word problems. They can solve word problems involving multiplication and division of whole numbers and fractions, add, subtract, and divide fractions with unlike denominators, and use pictorial representations of fractions in solving problems. Students at this level can order fractions having different denominators and decimals with differing numbers of decimal places. They also can write a decimal expressed in hundredths as a fraction in its lowest terms.

Students can select the correct rounding of a number involving four decimal places. They also can multiply a decimal expressed in thousandths by a decimal expressed in hundredths. Students can divide in decimal settings by a one-digit divisor expressed in thousandths. Similarly, they are able to divide a number involving one decimal place by a multiple of 100.

Students at this level are able to identify the ratio expressing a given whole number comparison in a word problem. They can solve multi-step word problems involving proportions.

Students at this level recognize that precision of measurement is related to the size of the unit of measurement. They can solve problems involving area and perimeter of rectangles, including problems that combine rectangles and triangles, and demonstrate understanding of the relationship between area and perimeter and length and perimeter.

Students have an elementary understanding of probability, including independent events. They can solve simple problems involving the relationships between successful and unsuccessful outcomes and probabilities. They can extrapolate a graph and read data from a frequency table.

Students at this level use their knowledge of supplementary and overlapping angles to solve problems. They know the properties of a parallelogram, can use knowledge of the properties of congruent triangles to solve problems, and can determine the number of triangles congruent with a given triangle needed to cover the area of a polygon. They can locate points in the first quadrant of the Cartesian plane and identify the image of a polygon under a half turn.

Students can recognize properties of operations on real numbers represented in symbolic form and recognize that multiplication can be used to represent repeated addition in algebraic expression. Students can identify algebraic expressions corresponding to verbal descriptions. They can substitute numbers for variables to find values of an expression and can subtract one algebraic expression from another with the same denominator.

Students can identify a linear equation that describes the relationship between two variables given by a table of values and can solve a linear equation with the variable on both sides of the equal sign. When given the first three terms of a sequence of terms pictorially, students can find a specified later term. Students can also solve problems involving terms common to two different arithmetic sequences. They can solve multi-step word problems when given a total and several conditions.

Achievement at the Upper Quarter International Benchmark

Figure 8 describes performance at the Upper Quarter benchmark for eighth grade. Students reaching this benchmark typically demonstrate success on the knowledge and skills represented by this benchmark, as well as the Median and Lower Quarter benchmarks. Eighteen percent of U.S. eighth-grade students scored at or above the Upper Quarter benchmark, below the international percentage.

Fractions and Decimals

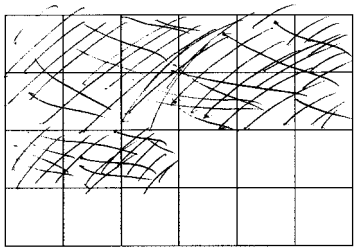
Students reaching the Upper Quarter benchmark demonstrate a strong grasp of fractions and decimals through computing, ordering, and rounding to solve word problems. Example items 11, 12, 13, 14, and 15 illustrate a range of ways in which students reaching this level use fractions and decimals successfully.

In example item 11, students shade in 5/8 of a figure with 24 squares. Although nearly all of Singaporean eighth graders (92 percent) correctly shaded in 15 of the 24 squares, only 43 percent of U.S. eighth graders did so. It was common (23 percent) for U.S. eighth graders to shade in only 5 squares, presumably not accounting for the fact that there are 24, not 8, squares in the grid. Mediocre performance by U.S. students on this item may be due more to them not thinking through the problem than about them not understanding fractions.

Grade 8 Example 11

Depicts an equivalent representation of a fraction by shading in squares in a rectangular grid.

Shade in $\frac{5}{8}$ of the unit squares in the grid.



N19

Percent Correct

★ United States
43

▲ Singapore
92

● International Avg.
51

Example 12 Grade 8

Adds three fractions with unlike denominators.

$\frac{3}{4}+\frac{8}{3}+\frac{11}{8}=$

A. $\frac{22}{15}$

B. $\frac{43}{24}$

C. $\frac{91}{24}$

(D.) $\frac{115}{24}$

K09

Percent Correct

★ United States
45

▲ Singapore
90

● International Avg.
49

In item 12, students add three fractions with unlike denominators. While 90 percent of Singaporean eighth graders added these correctly, less than half (45 percent) of U.S. eighth graders did so.

In example item 13, students write a decimal expressed in hundredths as a fraction in lowest terms. U.S. eighth graders did better than the international average (46 percent correct compared with 34 percent correct), but were outperformed by many countries, including top-performing Singapore (90 percent correct).

In example item 14, students solve a one-step word problem involving division of a whole number (5) by a unit fraction (1/4). About half of eighth-grade students internationally and in the United States determined that Luis runs the course 20 times a day, while 84 percent did so in Singapore.

Example 13 Grade 8

Writes decimal expressed in hundredths as a fraction in lowest terms.

Write 0.28 as a fraction reduced to its lowest terms.

Answer: $\frac{7}{25}$

P16

Percent Correct

★ United States
46

▲ Singapore
90

● International Avg.
34

Example 14 Grade 8

Solves a one-step word problem involving division of a whole number by a unit fraction.

Luis exercises by running 5 km each day. The course he runs is $\frac{1}{4}$ km long.

How many times through the course does he run each day?

Answer: 20

O09

Percent Correct

★ United States
51

▲ Singapore
84

● International Avg.
50

Grade 8 Example 15

Solves a one-step word problem involving division of a decimal by a whole number.

A stack of 200 identical sheets of paper is 2.5 cm thick. What is the thickness of one sheet of paper?

A. 0.008 cm

B. 0.0125 cm

C. 0.05 cm

D. 0.08 cm

R07

Percent Correct

★ United States
39

▲ Japan
75

● International Avg.
48

In example item 15, students solve a one-step word problem involving division of a decimal number (2.5) by a whole number (200). This item was a little more difficult than example item 14 for U.S. eighth graders (39 percent compared with 51 percent).

The Number and Operations Standard calls for grades 6-8 students to "work flexibly with fractions, decimals, and percents to solve problems" and "compare and order fractions, decimals, and percents efficiently…" The performance of U.S. eighth-grade students compared with that of students in the top-performing countries is disappointing given the emphasis in *Principles and Standards* on having grades 6–8 students work with fractions and decimals. This is an area that is expected to have been included in the mathematics instruction for students by the end of eighth grade.

Word Problems Involving Proportional Reasoning

Solving problems involving proportions is something that students reaching the Upper Quarter benchmark can typically do, and that students reaching lower benchmarks cannot typically do. In item 16, students solve a multi-step word problem involving proportions to determine the amount Sue paid. This item was difficult for U.S. eighth graders (23 percent correct).

Grade 8 Example 16

Solves a multi-step word problem involving whole number proportions.

Peter bought 70 items and Sue bought 90 items. Each item costs the same and the items cost $800 altogether. How much did Sue pay?

Answer: Sue paid $450

10 800 $160 x 5 = 800 90 x 50 = 450

R14

Percent Correct

★ United States
23

▲ Singapore
83

● International Avg.
38

Example 17 Grade 8

Applies knowledge of perimeter and area of a rectangle to solve a multi-step problem.

The length of a rectangle is 6 cm, and its perimeter is 16 cm. What is the area of the rectangle in square centimeters?

Answer: 12

K05

Percent Correct

★ United States 22

▲ Singapore 86

● International Avg. 39

Area and Perimeter

Students reaching the Upper Quarter benchmark have grasped the relationship between area and perimeter. This is illustrated by example item 17 in which students determine the area of a rectangle, given its length and perimeter. This item was difficult for U.S. eighth graders, with only 22 percent correctly determining that the area of the rectangle is 12 square centimeters. More U.S. students (25 percent) provided 96 (length × perimeter) as the area. It is disappointing that so many U.S. students would fail to grasp the relationship between area and perimeter, as this is addressed in the NCTM Geometry Standard for grades 6–8, which calls for students to "understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects."

Probability

Students reaching the Upper Quarter benchmark have an elementary understanding of probability and can solve simple problems involving the relationship between successful and unsuccessful outcomes and probabilities. For example, in item 18, students apply the definition of probability to determine that in a jar with 9 chips numbered 1 through 9, Madeline will draw an even-numbered chip 4/9 of the time.

U.S. eighth graders performed well on example item 18 (74 percent correct), significantly above the international average of 54 percent correct and not too far from the two top-performing countries, Korea (90 percent correct) and Japan (82 percent correct). *Principles and Standards* calls for instructional programs to begin addressing probability in grades 3–5. For example, students are expected to "describe events as likely or unlikely…" and "predict the probability of outcomes of simple experiments and test the predictions." In grades 6–8, students are expected to go beyond that to "understand and use appropriate terminology to describe complementary and mutually exclusive events," "use proportionality and a basic understanding of probability to make and test conjectures…" and "compute probabilities for simple compound events…".

Grade 8 Example 18

Percent Correct

★ United States
74

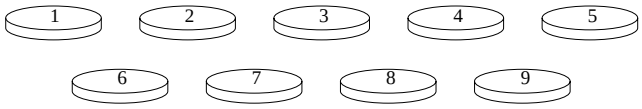
▲ Korea
90

▲ Japan
82

● International Avg.
54

Applies the definition of probablility to solve a word problem.

The nine chips shown are placed in a jar and mixed.



Madeleine draws one chip from the jar. What is the probability that Madeleine draws a chip with an even number?

A. $\frac{1}{9}$

B. $\frac{2}{9}$

(C.) $\frac{4}{9}$

D. $\frac{1}{2}$

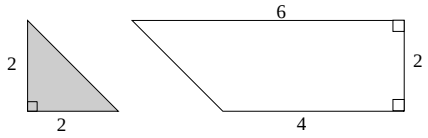
N18

Congruent Triangles and Angles

In geometry, students reaching the Upper Quarter benchmark for eighth grade use their knowledge of geometric properties to solve problems. These students have expanded their understanding of geometric figures beyond that at the Median benchmark (see below) to congruent triangles. For example, in item 19 students determine how many triangles of the same size and shape as the given triangle are needed to cover the area of the polygon shown. U.S. eighth graders (44 percent correct) performed below the international average (52 percent correct) and well below top-performing Japan (84 percent correct). A quarter of U.S. eighth graders identified that the trapezoid could be divided into five triangles.

Grade 8 Example 19

Solves a problem involving determining the number of triangles, congruent with a given triangle, needed to cover area of given polygon.



How many triangles of the shape and size of the shaded triangle can the trapezoid above be divided into?

A. Three

B. Four

(C.) Five

D. Six

R10

Percent Correct

★ United States
44

▲ Japan
84

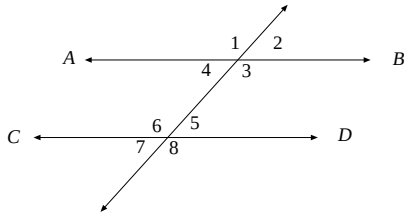
● International Avg.
52

The content assessed in example item 19 aligns with the expectation (Geometry Standard) that students in grades 3–5 "investigate, describe, and reason about the results of subdividing, combining, and transforming shapes." U.S. eighth graders should have a stronger grasp of congruent triangles than exhibited by their performance on this item.

Example 20 Grade 8

Uses knowledge of angles formed by parallel lines cut by a transversal.

In this figure, lines *AB* and *CD* are parallel.



Two angles whose measures must add up to 180° are

A. ∠1 and ∠3

B. ∠4 and ∠6

C. ∠2 and ∠5

D. ∠2 and ∠7

E. ∠1 and ∠8

O03

Percent Correct

★ **United States** 33

▲ **Japan** 83

● **International Avg.** 47

At the Upper Quarter benchmark, students have expanded their understanding of straight angles (see Median benchmark for eighth grade, below) to supplementary and overlapping angles. For example, in example item 20, students identify the two angles that must add up to 180 degrees. Only about one-third (33 percent) of U.S. eighth graders correctly answered this item, while nearly half (47 percent) of students internationally and 83 percent of students in Japan did.

Describing and relating angles is including in the NCTM Geometry Standard for grades 6–8, which calls for students to "precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties" and "understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects."

Algebraic Expressions and Linear Equations

In example item 21, students identify the algebraic expression that represents the verbal description of the number of hats Clarissa has. In the United States, 49 percent of eighth graders identified the correct expression, indicating that about half of eighth-grade students have difficulty with traditional algebra items such as this.

Grade 8 Example 21

Identifies algebraic expressions corresponding to a verbal description.

Juan has 5 fewer hats than Maria, and Clarissa has 3 times as many hats as Juan. If Maria has n hats, which of these represents the number of hats that Clarissa has?

A. $5 \pm 3n$

B. $3n$

C. $n \pm 5$

D. $3n \pm 5$

(E.) $3(n \pm 5)$

Q01

Percent Correct

★ United States 49

▲ Singapore 86

● International Avg. 46

While at the Median benchmark eighth-grade students solve equations with one variable (see below), students reaching the Upper Quarter benchmark solve equations with the variable on each side of the equation, demonstrating an increased grasp of solving for unknown values. To illustrate, in example item 22, students gather like terms and solve for x. Less than one-third (31 percent) of U.S. eighth graders solved this problem correctly. In contrast, 86 percent of Japanese eighth graders determined that $x = 7$.

Representing an unknown quantity with a letter or symbol is a concept that is to be introduced in grades 3–5, according to *Principles and Standards*, and using variables in expressions or equations receives emphasis in the Algebra Standard for grades 6–8. The percentage of U.S. students having difficulty with items like examples 21 and 22 indicates that these concepts are not well addressed in the classroom.

Grade 8 Example 22

Percent Correct

★ United States 31

▲ Japan 86

● International Avg. 45

Solves a linear equation with the variable on each sides of the equation.

Find x if $10x - 15 = 5x + 20$

Answer: 7

L16

FIGURE 9 Grade 8

Median International Benchmark — Eighth Grade

Use understanding of rounding in problem situations; perform basic operations with familiar fractions; understand place value of decimal numbers; understand measurement in several settings; locate data in charts and graphs to solve word problems; know and use simple properties of geometric figures to solve problems; identify algebraic expressions and solve equations with one variable.

Students at the Median benchmark can use their understanding of rounding in problem situations. They can produce an example of a number that would round to a given value [V01] and use rounding to estimate products.

Students can apply order of operations to add and multiply fractions. They can identify equivalent fractions and the smallest fraction in a list of fractions. Students recognize models representing fractions given in shaded regions and can use logical reasoning with a fraction. They can use their knowledge of ordering fractions to compare the likelihoods of several events.

Students at this level demonstrate an understanding of place value in decimal numbers. Given an object of one length (to one decimal place), they can estimate the length of another object. Given a length rounded to the nearest centimeter, they can identify an example of the actual length expressed to one decimal place. They can estimate the location of a point representing a decimal number with one decimal place on a whole number line, and they can order decimal numbers with different numbers of decimal places. They can set up and solve one-step problems involving addition and subtraction of numbers having two or fewer decimal places, including situations where the numbers have a different number of decimal places.

Students are able to set up and solve problems involving whole number proportions and problems involving interpretations of the ratio of parts to a total.

Students at this level demonstrate their understanding of measurement in several settings. They can compare volumes by visualizing and counting cubes. Given the start time and duration of an event, they can determine the end time. They recognize the inverse relationship between the size of a unit and the number of units required to measure the length of an object. They can estimate the distance between two points on a map, given the scale.

Students can solve word problems that require them to locate and interpret data in bar graphs as well as linear and non-linear graphs.

Students can identify simple properties of geometric figures and use this knowledge to solve problems (e.g., lines of symmetry, rotations of two-dimensionaland three-dimensional figures, straight angles, and congruent and similar triangles). Additionally, they can visualize the arrangement of faces of a cube when shown its net and can locate a point on a line given its distance from two other points on the line.

Students at this level recognize the relationship between exponents and repeated multiplication. Students can select an expression to represent a situation involving multiplication, identify a linear equation corresponding to a given verbal statement, and identify the linear expression corresponding to a table of ordered pairs. Students can substitute in a formula to determine the value of a variable and can solve for x given a linear equation of the form a(x+b)=c.

Achievement at the Median International Benchmark

Figure 9 describes performance at the Median benchmark for eighth grade. Students reaching this benchmark typically demonstrate success on the knowledge and skills represented by this benchmark, as well as the Lower Quarter benchmark. Forty-five percent of U.S. eighth-grade students scored at or above the Median benchmark, below the international percentage.

Fractions and Decimals

Students reaching the Median benchmark demonstrate a grasp of fractions. They identify equivalent fractions, recognize models representing fractions given in shaded regions, and can use logical reasoning with a fraction. In addition, they can perform basic operations with familiar fractions. For example, in item 23, students add and multiply fractions, applying the rules of operations. Similar to the international average, only half of U.S. eighth graders selected the correct answer for this straightforward problem.

In example item 24, students identify the set of fractions in which all of the fractions are equivalent. About two-thirds (67 percent) of U.S. eighth graders correctly answered this item, comparable to the international average, while 90 percent of students in Singapore and Belgium (Flemish) did so.

Grade 8 Example 23

Adds and multiplies fractions by applying order of operations.

$$\frac{3}{4}+\left(\frac{2}{3}\times\frac{1}{4}\right)=$$

A. $\frac{1}{8}$

B. $\frac{5}{16}$

C. $\frac{17}{48}$

D. $\frac{5}{6}$

(E.) $\frac{11}{12}$

Q09

Percent Correct

★ United States
50

▲ Singapore
89

● International Avg.
51

Grade 8 Example 24

Identifies a set of equivalent fractions.

In which list of fractions are all of the fractions equivalent?

A. $\frac{3}{4}, \frac{6}{8}, \frac{12}{14}$

B. $\frac{3}{5}, \frac{5}{7}, \frac{9}{15}$

(C.) $\frac{3}{8}, \frac{6}{16}, \frac{12}{32}$

D. $\frac{5}{10}, \frac{10}{15}, \frac{1}{2}$

N14

Percent Correct

★ United States
67

▲ Belgium (Fl)
90

▲ Singapore
90

● International Avg.
66

Students reaching the Median benchmark work with decimal numbers and demonstrate understanding of place value for decimals. For example, in item 25 students solve a word problem by adding decimals with unequal numbers of decimal places. While 71 percent of U.S. eighth graders correctly solved this problem, more than one-quarter were unable to do so.

Example 25 Grade 8

Solves a one-step addition word problem involving numbers with differing numbers of decimal places.

A chemist mixes 3.75 milliliters of solution A with 5.625 milliliters of solution B to form a new solution. How many milliliters does this new solution contain?

Answer: 9.375

3.75
5.625
9.375

K02

Percent Correct

★ United States 71

▲ Singapore 92

● International Avg. 65

Example 26 Grade 8

Given a weight (mass) rounded to the nearest 10 kg, can produce an example of actual weight (mass).

Rounded to the nearest 10 kg the weight of a dolphin was reported as 170 kg. Write down a weight that might have been the actual weight of the dolphin.

Answer: 168 (one possible answer)

V01

Percent Correct

★ United States 66

▲ Singapore 89

● International Avg. 54

Rounding

Students reaching the Median benchmark demonstrate that they understand rounding and can use it to solve problems. For example, in item 26 students produce an actual value (weight), given a rounded value. U.S. eighth-grade students performed above the international average on this item.

Measurement

Students reaching the Median benchmark demonstrate their understanding of measurement in a variety of settings, including measuring time. In example item 27, students determine when the cake should be taken out of the oven, given when it went in and how long it needs to bake. U.S. eighth graders performed below the international average on this relatively straightforward word problem. The Netherlands and Hungary were the two highest-performing countries, with 91 and 90 percent correct, respectively.

Grade 8 Example 27

Given the start time and duration of an event, determines the end time.

A cake is put in the oven at 7:20. If the cake takes three quarters of an hour to bake, at what time should it be taken out of the oven?

Answer: 8:05

O06

Percent Correct

★ **United States** 56

▲ **Netherlands** 91

▲ **Hungary** 90

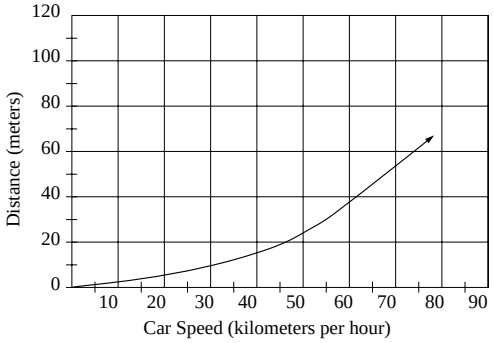
● **International Avg.** 69

Using Data in Charts and Graphs to Solve Word Problems

Example 28 Grade 8

Solves a word problem by reading information from a graph of a non-linear relationship.

The graph shows the distance traveled before coming to a stop after the brakes are applied for a typical car traveling at different speeds.



A car traveling on a highway stopped 30 m after the brakes were applied. About how fast was the car traveling?

A. 48 km per hour

B. 55 km per hour

C. 70 km per hour

D. 160 km per hour

O01

Percent Correct

★ United States
72

▲ Belgium (Fl)
82

▲ France
81

● International Avg.
59

Students reaching the Median benchmark are able to read and use data in bar graphs as well as linear and non-linear graphs. While these visual displays of data are not complicated and the tasks are not difficult, they are more complex than those successfully completed by students not reaching this level. An example of the kind of graph-reading task that students at this benchmark can do is item 28, in which students read information in a graph of a non-linear relationship of distance and speed to determine the speed at which the car was traveling before its brakes were applied. U.S. eighth-graders performed relatively well on this item, above the international average and close to highest-performing Belgium (Flemish) and France.

Geometry

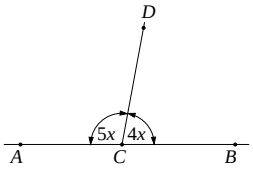
Students reaching the Median international benchmark know and use simple properties of geometric figures to solve problems. Example items 29 and 30 illustrate students' understanding of straight angles and rotation.

To answer example item 29 correctly, students know that a straight angle is 180 degrees and form an equation to solve for the measure of an angle. Sixty-one percent of U.S. eighth graders answered this correctly, while 92 percent did so in highest-performing Singapore. This item aligns with the expectation (Geometry Standard) that students "precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties." Students' understanding of angles goes beyond straight angles to supplementary and complementary angles at the Upper Quarter benchmark (see above).

Grade 8 Example 29

Applies knowledge that a straight angle has 180 degrees to form an equation and solves for the measure of an angle.

In this figure *AB* is a straight line.



What is the measure, in degrees, of angle *BCD* ?

A. 20

B. 40

C. 50

D. 80

E. 100

M07

Percent Correct

★ **United States** 61

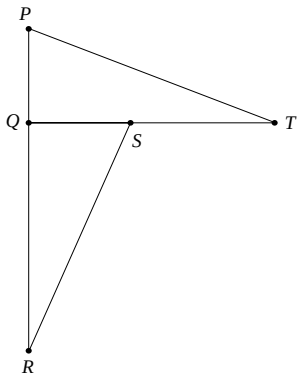
▲ **Singapore** 92

● **International Avg.** 72

Example 30 Grade 8

Given a figure and its image under a rotation, identifies the center of rotation.

Triangle *PQT* can be rotated (turned) onto triangle *SQR*.



What point is the center of rotation?

A. *P*

B. *Q*

C. *R*

D. *S*

E. *T*

O08

Percent Correct

★ **United States**

65

▲ **Japan**

93

● **International Avg.**

70

In item 30, students identify the center of rotation of a triangle, given the figure and its image under rotation. This item was very easy for Japanese eighth graders (93 percent correct), while only 65 percent of U.S. eighth graders successfully identified *Q* as the point of rotation. In the United States, the most popular incorrect response (20 percent) was *S*.

This item aligns with NCTM expectation (Geometry Standard) that grades 6–8 students "describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling."

Algebraic Expressions and Linear Equations

Students reaching the Median benchmark have developed the basic understanding of working with variables and can deal with algebraic expressions and linear equations of the form a(x+b)=c. For example, in example item 31, students identify that $y \times y \times y$ is the same as y^3. Seventy-four percent of U.S. eighth graders answered this correctly. This item aligns with the NCTM Algebra Standard for grades 6–8, which calls for students to "relate and compare different forms of representation for a relationship."

In item 32, students solve for x in a fairly straightforward linear equation. Seventy-three percent of U.S. eighth graders answered this correctly. This item aligns with the NCTM Algebra Standard for grades 6–8, which calls for students to "use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships."

Grade 8 Example 31

Recognizes that an exponent indicates repeated multiplication.

Which of these expressions is equivalent to y^3 ?

A. $y + y + y$

B. $y \times y \times y$

C. $3y$

D. $y^2 + y$

P15

Percent Correct

★ United States 74

▲ Singapore 91

● International Avg. 66

Grade 8 Example 32

Solves a linear equation involving parentheses.

If $3(x + 5) = 30$, then $x =$

A. 2

B. 5

C. 10

D. 95

O07

Percent Correct

★ United States 73

▲ Singapore 96

● International Avg. 72

FIGURE 10 Grade 8

Lower Quarter International Benchmark — Eighth Grade

Understand different representations of fractions – verbal and decimal; add and subtract decimals with the same number of decimal places; read, locate, and compare data in charts and graphs; calculate average of whole numbers.

Students at the Lower Quarter benchmark demonstrate familiarity with fractions and decimals. They can write a fraction larger than a given fraction and translate between verbal and numerical representations of a number with one decimal place. They can add and subtract decimals involving the same number of decimal places with multiple regroupings. Students can round whole numbers to the nearest hundred and can subtract whole numbers involving multiple regroupings and zeroes.

Students are able to select the appropriate metric unit to measure the mass of an object. They can read a scale with unlabeled tick marks, and they can estimate the size in degrees of an acute angle.

Students can read, locate, and compare data in tables, bar graphs, circle graphs, and pictographs. Students can find the missing term in a proportion and have an intuitive understanding of the likelihood of an event. They can calculate the average of a set of whole numbers.

Students are able to recognize a pattern of geometric shapes presented in a different visual format. Using the properties of a balance, they can reason to find an unknown weight .

Achievement at the Lower Quarter International Benchmark

Figure 10 describes performance at the Median benchmark for eighth grade. Students reaching this benchmark typically demonstrate success on the knowledge and skills represented by this benchmark. Seventy-five percent of U.S. eighth-grade students scored at or above the Lower Quarter benchmark, matching the international percentage.

Fractions and Decimals

Students reaching only the Lower Quarter benchmark demonstrate very little in the way of competence with fractions and decimals. They do understand verbal and decimal representations of fractions and can add and subtract decimal numbers with the same number of decimal places. They tend to be successful on straightforward problems involving fractions and decimals, yet are typically not successful on word problems involving fractions and decimals.

In example item 33, students identify the numerical representation of the number expressed verbally in the question. This was a very easy question for U.S. eighth graders (91 percent), and for eighth-grade students in the Czech Republic and the Slovak Republic, where nearly all eighth-grade students provided the correct answer. Given that recognizing equivalent representations of numbers is addressed in the Number and Operations Standard for grades 3–5, calling for students to "recognize equivalent representations for the same number…," U.S. eighth graders should be performing well on this type of item.

Grade 8 Example 33

Translates between verbal and numerical representation to one decimal point.

Which number is five hundred four and seven-tenths?

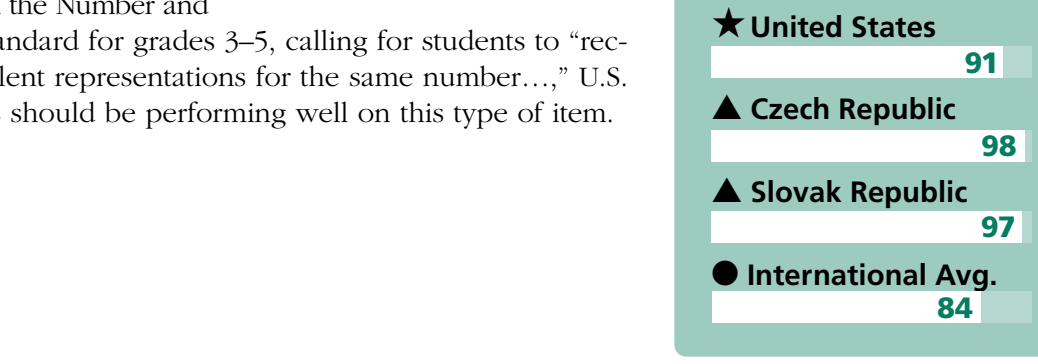
A. 54.7

B. 504.7

C. 547

D. 5004.7

L09



Example 34 Grade 8

Subtracts to three decimal points using multiple regrouping.

Subtract: $2.201 \pm 0.753 =$

(A.) 1.448

B. 1.458

C. 1.548

D. 1.558

R06

Percent Correct

★ United States 74

▲ Czech Republic 90

▲ Hungary 90

● International Avg. 75

In example item 34, students subtract one decimal number from another; both have the same number of decimal places. The Czech Republic and Hungary were the highest-performing countries (both with 90 percent correct). In the United States and internationally, about a quarter of eighth graders could *not* do this rather straightforward computation problem (74 percent correct).

Rounding Whole Numbers

Students reaching the Lower Quarter benchmark can round whole numbers to the nearest hundred, as illustrated in example item 35 in which students identify the actual number of trees planted, given the rounded number reported in a newspaper. This item was quite easy for U.S. eighth graders (88 percent correct).

Example 35 Grade 8

Given a five-digit number rounded to the nearest hundred, can identify what the original number could have been.

A newspaper reported that about 18 200 trees had been planted in the park. The number was rounded to the nearest hundred. Which of these could have been the actual number of trees planted?

A. 18 043

(B.) 18 189

C. 18 289

D. 18 328

N11

Percent Correct

★ United States 88

▲ Singapore 98

● International Avg. 82

Measurement

In measurement, students at this benchmark demonstrate rather rudimentary understanding and little application to problem solving. For example, in example item 36 students read the value of an unlabeled tick mark on the drawing of a scale and determine the weight shown. U.S. eighth graders performed the same as eighth graders internationally (86 percent correct), and nearly all students in Belgium (Flemish) and the Netherlands answered this item correctly. Given that this measurement skill is included in the NCTM Measurement Standard for grades 3–5, U.S. eighth graders should do well on this type of item.

Grade 8 Example 36

Reads the value indicated by an unlabeled tick mark on a circular scale.

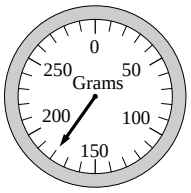
What is the weight (mass) shown on the scale?

A. 153 g

B. 160 g

C. 165 g

D. 180 g



M01

Percent Correct

★ **United States** 86

▲ **Belgium (Fl)** 98

▲ **Netherlands** 97

● **International Avg.** 86

Example 37 Grade 8

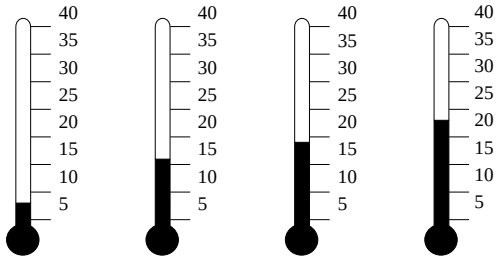
Locates a piece of information in a table and matches reading on thermometer.

This table shows temperatures at various times during the week.

TEMPERATURES					
	6 a.m.	9 a.m.	Noon	3 p.m.	8 p.m.
Monday	15	17	20	21	19
Tuesday	15	15	15	10	9
Wednesday	8	10	14	13	15
Thursday	8	11	14	17	20

Which thermometer shows the temperature at 8 p.m. on Monday?

A. B. (C.) D.



P17

Percent Correct

★ **United States** 80

▲ **Switzerland** 97

▲ **France** 97

● **International Avg.** 82

Reading Data in Graphs

Students reaching the Lower Quarter benchmark are successful at fairly basic graph reading. For example, in item 37, students locate the temperature on Monday at 8pm in the table and match it to the thermometer. In Switzerland and France, nearly all eighth-grade students (97 percent) identified the thermometer showing the temperature on Monday. Graph reading is an area in which U.S. students typically did well on TIMSS, and 80 percent did find the correct thermometer. However, 20 percent of U.S. eighth graders did not answer this very straightforward matching task correctly.

In example item 38, students use the verbal description of four girls' heights to determine which girl is represented by each bar in the graph, and then read the bar graph to determine Sarah's height. With 88 percent correct, U. S. eighth graders performed close to the two top-performing countries on this item, Ireland (94 percent correct) and Singapore (93 percent correct).

Grade 8 Example 38

Percent Correct

★ United States 88

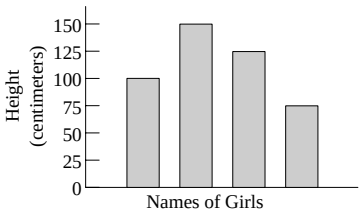
▲ Ireland 94

▲ Singapore 93

● International Avg. 83

Solves a comparison problem by associating elements of a bar graph with a verbal description.

The graph shows the heights of four girls.



The names are missing from the graph. Debbie is the tallest. Amy is the shortest. Dawn is taller than Sarah. How tall is Sarah?

A. 75 cm

B. 100 cm

C. 125 cm

D. 150 cm

Q04

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Appendix

Descriptions of Items at Each Benchmark: Fourth- and Eighth-Grades

Appendix

Descriptions of Items at Each Benchmark* Grade 4

Lower Quarter Benchmark Items

Whole numbers	
A04	Recognizes a pictorial representation of ones, tens, and hundreds and can identify the standard numeral
B08	Translates between standard and expanded notation of three-digit whole numbers
C04	Recognizes multiplication as the appropriate operation in a one-step word problem (single-digit)
E02	Recognizes the hundreds place in a four-digit number
G03	Translates between a numeric and verbal representation of a four-digit number
I04	Multiplies a two-digit number by a single-digit number with no regrouping (problem uses the word "times")
J09	Recognizes and extends a number pattern in a hundreds chart
K02	Adds two four-digit numbers involving three regroupings
M08	Identifies the largest four-digit number formed by four different, given, digits
H07	Recognizes inequality symbols and can choose the largest of two three-digit numbers
H09	Understands tens place value and can translate between verbal and numeric representations
U05	Represents simple repeated addition as multiplication

* Alphanumeric code corresponds to the item cluster in which the item appeared, and the order with the cluster.

Item Descriptions Grade 4

Lower Quarter Benchmark Items *continued*

Fractions and proportionality	
A01	Recognizes one-half of a set of objects

Measurement, estimation and number sense	
A05	Compares areas by counting squares
K05	Estimates length of short object in centimeters

Data representation, analysis, and probability	
C01	Reads information from a simple bar graph
D05	Reads information from a simple bar graph
J03	Solves a simple addition problem based on information in a pie graph (%)
M01	Knows that there is a better chance of landing on a shaded part of an area when a greater part of that area is shaded
T01A	Reads an unlabeled scale point on a simple bar graph

Geometry	
D07	Identifies two figures that have the same size and shape
F09	Knows that every triangle has three sides
I06	Recognizes the difference between straight and curved sides of a figure

Item Descriptions Grade 4

Geometry *continued*	
J01	Recognizes the figure Δ is a triangle
L03	Identifies an object at a given location in a simple, two-dimensional coordinate system

Algebra	
H08	Identifies next terms in an alternating number pattern involving counting forward and backward by ones
L04	Recognizes same pattern of sequence of shapes when made with different shapes

Median Benchmark Items

Whole numbers	
I09	Subtracts one four-digit number from another in a problem involving multiple regrouping and 0s
S02	Provides missing term in expanded notation of four-digit number
V03	Solves a problem counting backwards or subtracting across a hundred point
A03	Solves a one-step word problem involving multiplication of a three-digit by a one-digit number requiring regrouping
F07	Translates from a form of expanded notation to a standard notation for a five-digit number

Item Descriptions Grade 4

Median Benchmark Items *continued*

Whole numbers *continued*	
M03	Recognizes that the order of two numbers in multiplication problem involving two numbers does not affect the answer
M06	Solves two-step word problem involving comparison of whole numbers

Fractions and proportionality	
B09	Adds decimal numbers involving tenths
C03	Recognizes a familiar fraction represented by a figure with shaded parts (region model)
D08	Subtracts two decimals involving hundredths with regrouping over 0
J07	Recognizes what fraction of a rectangular figure is shaded
U02	Writes a fraction larger than a given fraction (2/7)

Measurement, estimation, and number sense	
C02	Calculates the volume of a rectangular solid given the volume of one layer and the number of layers
E03	Rounds a three-digit whole number to the nearest hundred
F08	Selects appropriate metric unit to measure weight (mass)
E01	Counts weeks forward from a given date on a calendar

Item Descriptions Grade 4

Measurement, estimation, and number sense *continued*	
G02	Selects a reasonable metric weight (mass) for an adult
J06	Selects the largest metric unit of weight (mass)

Data representation, analysis, and probability	
L01	Interprets the numerical value of a symbol on a pictograph
B05	Solves a comparison problem by associating elements of a bar graph with a verbal description
F05	Locates data in a two-way table

Geometry	
T05	Given half of a symmetric figure, draws the complete figure
B06	Recognizes flat and curved surfaces on solids
J02	Recognizes line of symmetry
K01	Shows understanding of logical connectives "and" and "not" in locating a point in a region in a figure of overlapping triangle, square, and circle
K08	Identifies four equal parts in a rectangle

Median Benchmark Items *continued*

Algebra	
K06	Finds a specified term of a sequence given the first three terms pictorially
I07	Recognizes the number sentence used to represent a word problem involving addition or subtraction
L09	Given two greater than relationships involving three quantities, determines the third relationship
M09	Identifies a whole number that satisfies a simple inequality involving multiplication

Upper Quarter Benchmark Items

Whole numbers	
T02	Orders four digits to make the smallest possible four-digit whole number
I03	Solves two-step word problem involving subtraction
L07	Finds the pair of numbers that differ by 100
V02	Writes a number that is 1,000 more than a given five-digit number
V04B	Arranges four digits into two two-digit numbers whose sum is greater than a given number (in the context of a game)

Item Descriptions Grade 4

Fractions and proportionality	
S04	Solves a multi-step problem involving a rate in half-hour time intervals
U03B	Solves one-step multiplication problem involving rate
A02	Solves two-step word problem using doubling and adding
D09	Recognizes the figure that illustrates a simple ratio
F06	Solves word problem involving 1/2 and 1/4

Measurement, estimation, and number sense	
D06	Finds the increase in temperature from a negative to a positive temperature on a thermometer
T03	Finds the time an event started given its duration and end time
B07	Selects the number sentence that provides the best estimate of which is closest to the actual product of two two-digit numbers
L06	Given the mass of an object to one decimal place, estimates the mass of 1,000 of these objects
J08	Identifies the expression that is the best estimate adding three two-digit numbers
S05	Estimates the length of an object using a non-standard unit
U01	Finds the number of triangular tiles needed to cover a region

Upper Quarter Benchmark Items *continued*

Data representation, analysis, and probability	
K04	Solves a multi-step problem involving addition and subtraction of whole numbers in a table
M02	Solves a multi-step problem involving addition and subtraction of whole numbers in a table
T01B	Reads data from labeled and unlabeled points on a bar graph to find the total number of objects represented (shows work)
L02	Given a chance situation with one possible successful outcome, recognizes that the chance of success decreases as the number of equally likely unsuccessful outcomes increases
S01	Uses numerical data from a table to draw sets of double bars to complete a bar graph

Geometry	
I01	Locates a point on a rectangular grid by following specified moves
M04	Given the coordinates of a dot on a grid, determines the coordinates of another dot on the grid

Algebra	
U04	Finds the next number in a decreasing arithmetic sequence
G04	Selects the expression that represents a situation involving multiplication
K03	Selects the number pairs satisfying a specified number operation rule involving multiplication

Item Descriptions Grade 4

Top 10% Benchmark Items

Whole numbers	
J04	Finds difference between the products of two expressions having the same multiplier

Fractions and proportionality	
K09	Solves a two-step word problem involving division and multiplication by one-digit numbers
S03	Solves a word problem involving subtraction of decimals to one decimal place (tenths)
V01	Compares two unit fractions and shades part of a circle to justify the answer
E04	Identifies the decimal representation for a fraction with a denominator of 10
G01	Estimates the distance on a map given scale (in cm = km)
I02	Identifies the verbal representation for a common fraction that is equivalent to a numerical representation of a decimal number given in tenths
I05	Uses proportional reasoning to solve a word problem involving halves
I08	Recognizes pictorial representations of a pair of equivalent fractions
M05	Identifies decimal representation for shaded portion of a rectangular figure divided in 10 equal parts

Top 10% Benchmark Items *continued*

Measurement, estimation, and number sense	
H06	Identifies the numerical expression that gives the distance around a rectangle, given its length and width
M07	Identifies the quantity that would most likely be measured in milliliters
V05	States the number of millimeters in a meter

Geometry	
H05	Recognizes the equivalent of a three-dimensional figure when it is rotated to a different orientation

Algebra	
J05	Identifies the number rule involving division that describes the relationship between pairs of whole numbers given in a table

Item Descriptions Grade 4

Items Above the Top 10% Benchmark

Fractions and proportionality	
T04A	Shows that a ratio of 10:20 is equivalent to 1:2 using words or pictures
T04B	Selects and uses the appropriate quantities to compare a ratio to a fraction

Measurement, estimation, and number sense	
K07	Identifies the length of a rectangle given its perimeter and width
L08	Recognizes the inverse relationship between size of unit and number of units required to cover a distance

Geometry	
L05	Determines the number of edges of a cube with some edges not shown

Descriptions of Items at Each Benchmark* Grade 8

Lower Quarter Benchmark Items

Fractions and number sense	
H09	Rounds to estimate the sum of two three-digit numbers
I06	Writes a fraction larger than a given fraction (2/7)
L09	Translates between verbal and numerical representation to one decimal point
N11	Given a five-digit number rounded to the nearest hundred, can identify what the original number could have been
R06	Subtracts to three decimal points using multiple regrouping
R12	Subtracts one four-digit number from another in a problem involving multiple regrouping and zeroes

Proportionality	
B08	Solves a word problem by finding the missing term in a proportion

Measurement	
D11	Selects appropriate metric unit to measure weight (mass)
M01	Reads the value indicated by an unlabeled tick mark on a circular scale
N15	Identifies an angle closest to a given measurement in degrees

* Alphanumeric code corresponds to the item cluster in which the item appeared, and the order with the cluster.

Item Descriptions Grade 8

Data representation, analysis, and probability	
A06	Calculates and compares the averages of two sets of data
J13	Completes a pictogram involving part symbols from data given in the table
L10	Locates data in a two-way table
M03	Given a chance situation with one possible successful outcome, recognizes that the chance of success decreases as the number of equally likely unsuccessful outcomes increases
P17	Locates a piece of information in a table and matches reading on thermometer
Q04	Solves a comparison problem by associating elements of a bar graph with a verbal description
C02	Reads and interprets information from a pie graph

Algebra	
L13	Recognizes same pattern of sequence of shapes when made with different shapes
A02	Using properties of a balance, reasons to find an unknown weight (mass)

Median Benchmark

Fractions and number sense	
D09	Selects the smallest fraction from a set of familiar fractions
F09	Identifies a decimal number given in thousandths between two decimal numbers given in hundredths
J17	Estimates the distance on a map given scale (in cm = km)
K02	Solves a one-step addition word problem involving numbers with differing numbers of decimal places
P13	In a word problem, estimates a product of a two-digit number and multiple of 10
A01	Finds 4/5 of a region divided into 10 equal parts
B09	Given two equivalent fractions, identifies the pictorial representation showing they are equivalent
G05	Selects a fraction representing the comparison of part to whole, given each of two parts in a word problem setting
L08	Given the height of one object to one decimal place, identifies the estimate of the height of another
Q09	Adds and multiplies fractions by applying rules of order of operations
V01	Given a weight (mass) rounded to the nearest 10kg, can produce an example of actual weight (mass)
D12	Estimates the value, to one decimal place, of a point on a number line marked at whole number intervals
E04	Arranges four given digits in ascending and descending order and finds the difference between those two numbers

Item Descriptions Grade 8

Fractions and number sense *continued*	
H08	Selects a figure with shaded parts that represents a familiar fraction
I05	Solves one-step subtraction word problem involving two numbers with decimals to the hundredth
I07	Decides whether estimate or exact value is appropriate in a situation involving money
K01	Identifies a circular model of a fraction that best approximates a given rectangular model of the same fraction
N14	Identifies a set of equivalent fractions
P12	Selects the number sentence that provides the best estimate of which is closest to the actual product of two two-digit numbers
P14	Solves word problem involving 1/2 and 1/4

Proportionality	
Q05	Solves problem involving logical reasoning with properties of a fraction
D08	Solves for missing number in a proportion
G04	Solves equation for missing number in a proportion

Measurement	
A03	Given a length rounded to the nearest centimeter, identifies what the actual length could have been in centimeters to one decimal place
L12	Recognizes the inverse relationship between size of unit and number of units required to cover a distance

Median *continued*

Measurement *continued*	
C01	Compares volume by visualizing and counting cubes
O06	Given the start time and duration of an event, determines the end time
S02A	Solves a one-step problem involving division
G02	Identifies an unlabeled midway point on a number line marked in tenths

Data representation, analysis, and probability	
B07	Given two line graphs, identifies the relevant one and determines the interval showing the greatest increase
I09	Given a set of possible outcomes expressed as fractions of all outcomes, recognizes that the highest probability is associated with the largest fraction
H11	In a word problem, solves for a missing number in a proportion
O01	Solves a word problem by reading information from a graph of a non-linear relationship
E01	Solves problem by interpreting information from a graph of two intersecting lines
H07	Reads data on a bar graph to solve a word problem

Geometry	
A05	Identifies pairs of congruent triangles
B11	Visualizes the arrangement of the faces of a cube given its net

Item Descriptions Grade 8

Geometry *continued*	
C03	Identifies corresponding parts of congruent triangles
D07	Applies knowledge of symmetry to select the measure of an angle
J15	Identifies a pair of similar triangles
K03	Recognizes the equivalent of a three-dimensional figure when it is rotated to a different orientation
M02	Identifies lines of symmetry
M07	Applies knowledge that a straight angle has 180 degrees to form an equation and solves for the measure of an angle
N12	Locates a point on a number line given its distance from two other points
O08	Given a figure and its image under a rotation, identifies the center of rotation

Algebra	
B12	Identifies the linear equation corresponding to a given verbal statement involving a variable
P15	Recognizes that an exponent indicates repeated multiplication
Q07	Substitutes in a formula to determine the value of a variable
E05	Identifies the linear relationship between the first and second terms in a set of ordered pairs
H12	Selects the expression that represents a situation involving multiplication
O07	Solves a linear equation involving parentheses

Upper Quarter Benchmark Items

Fractions and number sense	
J14	Divides decimals involving thousandths
K09	Adds three fractions with unlike denominators
M04	Identifies largest of given fractions
P16	Writes decimal expressed in hundredths as a fraction in lowest terms
Q08	Orders a list of common fractions and decimal fractions in tenths, hundredths, and thousandths
R07	Solves a one-step word problem involving division of a decimal by a whole number
C04	Identifies the pair of numbers satisfying given conditions involving ordering integers, decimals, and common fractions
J12	Divides one fraction by another fraction
B10	Selects the smallest of a set of numbers with differing numbers of decimal places
E03	Identifies the fraction of an hour representing a time interval
F12	Identifies the interval containing the fraction that represents the shaded part of a circle
I02	Solves a multi-step word problem involving multiplication of a whole number by a fraction
L17	Subtracts fractions with unlike denominators
M08	Multiplies a decimal in the thousandths by a decimal in the hundredths

Item Descriptions Grade 8

Fractions and number sense *continued*	
N19	Depicts an equivalent representation of a fraction by shading in squares in a rectangular grid (5/8 of a 4x6 grid)
O04	Given a decimal number in four decimal places, rounds the decimal to the nearest hundredth
O09	Solves a one-step word problem involving division of a whole number by a unit fraction

Proportionality	
R14	Solves a multi-step word problem involving whole number proportions
V03	Identifies the ratio of part to total in a word problem
A04	Solves a word problem by finding the missing term in a proportion

Measurement	
E06	Identifies the length of a rectangle given its perimeter and width
J10	Solves a multi-step word problem involving areas of rectangles
K05	Applies knowledge of perimeter and area of a rectangle to solve a multi-step problem
V04	Finds the area of a parallelogram embedded in another figure
F10	Recognizes that precision of measurement is related to the size of the unit of measurement
P11	Solves a problem involving measuring and estimating length

Upper Quarter Benchmark Items *continued*

Data representation, analysis, and probability	
O05	In a word problem, when given the possible number of outcomes and the probability of successful outcomes, solves for the number of successful outcomes
R08	Solves a word problem by extrapolating a graph of a non-linear relationship
F08	Understands independence in a probability setting
G01	Reads data from a frequency table to solve a problem
K07	In a word problem, when given the possible number of outcomes and the probability of successful outcomes, solves for the number of successful outcomes
N18	Applies the definition of probability to solve a word problem

Geometry	
E02	Uses properties of congruent triangles to find the measure of an angle
J11	Knows properties of parallelogram
O03	Uses knowledge of angles formed by parallel lines cut by a transversal
Q10	Solves a problem involving measures of overlapping angles
G03	Solves a problem involving adjacent and vertical angles
J16	Identifies coordinates for a point in Cartesian plane
M05	Identifies image of a polygon under a rotation about a point

Item Descriptions — Grade 8

Geometry *continued*	
P08	Identifies ratio of length of side of square to its perimeter
R10	Solves a problem involving determining the number of triangles, congruent with a given triangle, needed to cover area of given polygon

Algebra	
F11	Solves a two-step problem involving multiplication and division of whole numbers and fractions
H10	Identifies the linear equation that describes the relationship between two variables given a table of values
I04	Solves a problem involving terms common to two arithmetic sequences
L16	Solves a linear equation with the variable on both sides of the equation
P10	Identifies algebraic expression showing repeated addition as multiplication
R09	Recognizes properties of operations on real numbers represented in symbolic form
R11	Solves multi-step word problem given total and several conditions
Q01	Identifies algebraic expressions corresponding to a verbal description
Q02	Subtracts algebraic fractions (e.g., x/5) with same denominator
C05	Finds a specified term in a sequence given the first three terms pictorially
G06	Finds the value of an algebraic expression involving multiplication of negative integers
N13	Finds the value of an algebraic expression in one variable by substituting a given value for the variable

Item Descriptions Grade 8

Top 10% Benchmark Items

Fractions and number sense	
N16	Solves multi-step problem with fractions requiring processing and organizing information
N17	Solves a word problem involving multiple operations with decimals
U01	Explains the strategy used to estimate the sum of time intervals given in minutes and seconds
O02	Calculates the percent of increase in price in a word problem
K06	Solves word problem involving the percent of increase and estimation
Q06	In a word problem, computes a product of a two-digit number (number of weeks per year) and multiple of 1000
R13	Solves a two-step problem involving multiplication of a whole number by a fraction

Proportionality	
F07	Solves a time-distance rate problem involving division of decimals and conversion of minutes to seconds
M06	Given the total number and the ratio between the two parts, finds the value of one part

Measurement	
Q03	Compares duration of times given in different units
S02C	Applies knowledge of the properties of squares and the relationship between perimeter and area to solve multi-step word problem

Item Descriptions — Grade 8

Measurement *continued*	
U02A	Given dimensions of a rectangle, draws a new rectangle whose dimensions are specified fractional multiples of the original dimensions
I03	Applies knowledge of number of milliliters in a liter to solve a word problem

Data representation, analysis, and probability	
V02	Selects, organizes, and processes relevant information from two advertisements to solve word problem involving whole numbers and different units

Geometry	
P09	Finds the length of corresponding sides of similar triangles
I08	Given the coordinates of two points on a line, identifies the coordinates of a third point
K08	Uses knowledge of congruence, corresponding parts, and the sum of degrees of the angles in a triangle to find the measure of an angle

Algebra	
D10	Identifies algebraic equation (formula) corresponding to a verbal description involving a constant and two variables
I01	Identifies what a variable in an equation represents in an equation involving the algebraic representation of consecutive whole numbers
J18	Finds and uses rule describing the relationship between ordered pairs given in a table
K04	Given a linear inequality involving fractions, identifies an equivalent form
T01	Given the sum and difference of two whole numbers in a word problem, shows how to determine the two numbers

Items Above the Top 10% Benchmark

Proportionality

T02A	Extracts information from a complex problem situation and uses a part and a ratio to find the total
T02B	Extracts information from a complex problem situation to determine both parts of a ratio

Measurement

U02B	Finds the areas of two rectangles and the ratio of their areas

Geometry

L15	Applies knowledge of the sum of measures of angles in a quadrangle to find the missing degree measure of an angle

Algebra

L11	Visualizes a physical situation presented in a word problem to determine the sequence and finds the sum of several terms
S01B	Uses algebraic reasoning to determine a pattern of square numbers and finds a specified term